TOPICS

sets, operations, relations, strings, closure of relation, countability and diagonalization, induction and proof methods, pigeon-hole principle, concept of language, formal grammars, chomsky hierarchy.

① **Sets:-** A set is a collection of elements without any structure or order. In other words a set may be defined as collection of objects and it is denoted by

(i) Enumerating the member within "{" and "}".

for example, $A = \{0,1,2\}$

$A = \{aa, aaaa, aaaaaa, ---\}$

The method by which the above sets are represented is called Roster or tabulation method. In this method the elements of the set are listed with in brackets ({ }) separated by commas {,}.

(ii) If every member of set A is a member of set B. then we write $A \subseteq B$, i.e A is subset of B.

(iii) set A and set B are equal if they have the same members

i.e $A = B$ iff (if and only if)

$A \subseteq B$ and $B \subseteq A$

(iv) if every member of ~~set B~~, A is in B and there is at least one member of B which is not there in A then we write

$A \subseteq B$ or $A \subset B$. A is called as proper subset of B

Cardinality:- The number of distinct elements in a set is called the cardinality of the set. For example there is a set A, having elements

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$A = \{0, 1, 2\}$

Number of distinct element in $A = 3$, There fore The Cardinality of set A is 3

Types of sets:-

(i) Equivalent set:- two finite sets A and B will be called equivalent sets if and only if they have The same cardinality. For example, sets A and B are equivalent sets represented as

$A = \{0, 1, 2\}$ $B = \{a, b, c\}$

(ii) Equal sets:- Two sets A and B will be said to be equal set if and only if they have the same elements For example, The set A and B represented as

$A = \{a, e, i, o\}$, $B = \{a, e, i, o\}$ are equal sets

(iii) Empty sets:- An empty set is defined as a set with no elements. It is also called a null set. It is denoted by ϕ. An empty set is a sub set of all sets. A set consisting of at least one element is called non-empty or non-void set.

(iv) Singleton sets:- A set containing only one element is called a singleton set

For example, The following sets are singleton sets:

$A_1 = \{0\}$, $A_2 = \{1\}$, $A_3 = \{x\}$

(v) Sub sets:- If every element in a set A is also an element of set B, Then A is called a subset of B It is written as

$A \subseteq B$ or $B \supseteq A$

The empty set is a sub set of every set.

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(vi) Power set: The power set of a set S is denoted by 2^S. If we have a set Q defined as $Q = \{a_1, a_2\}$, then the power set of Q has elements $\{\phi, \{a_1\}, \{a_2\}, \{a_1, a_2\}\}$. It is a set of all subsets of a given set

(vii) Super set: when A is a subset of B $(A \subseteq B)$ then B is a superset of A which is written as $B \supseteq A$. Let us consider sets A and B defined in the following ways.

(1) $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$, then $A \subseteq B$

(2) $A = \{\alpha, \beta, \gamma\}$ $B = \{\gamma, \beta, \alpha\}$, then $A \subseteq B$ and $B \subseteq A$

(VIII) Universal set:

A set U is called universal set if U is the super set of all the sets which are under our consideration. For example, If there sets A, B and U are defined as:

$A = \{1,2,3,5,6\}$

$B = \{4,7,8,9,10\}$

$U = \{1,2,3,4,5,6,7,8,9,10\}$

As we see that $A \subseteq U$ and $B \subseteq U$ Therefore U is the superset of the universal set and U is the super set of A and B.

[Dis]joint sets: Two sets A and B are said to be disjoint set if

$A \cap B = \{\phi\}$

For example, if set A and B are ~~said to~~ defined as

$A = \{a, b, c, d\}$, $B = \{A, B, C, D\}$

Then $A \cap B = \phi$

Therefore, A and B are disjoint sets.

② Operations :-

Operations on sets :-

Union :- The union of two sets A and B, is denoted by $A \cup B$. It is the set of elements which belong to A or B

For example,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$A = \{1,2,3,4\}$, $B = \{2,5,6,7\}$

$A \cup B = \{1,2,3,4,5,6,7\}$

Intersection :- The intersection of two sets A and B is denoted by $A \cap B$. It is the of elements which belong to both A and B.

For example.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A = \{1,2,4,5,7\}$, $B = \{2,4,5,8\}$

$A \cap B = \{2,4,5\}$

example :-

$A = \{a,b,c\}$, $B = \{0,1,2,3\}$, $C = \{A,B\}$

Then

$A \cup B \cup C = \{a,b,c,0,1,2,3,A,B\}$

and

$A \cap B \cap C = \{\phi\}$ i.e, an empty (or) null set

and

$A \cup B \cap C = \{a,b,c,0,1,2,3\}$

and

$A \cap B \cup C = \{A,B\}$

Set Difference:- If A and B are the two sets then the difference of A and B denoted as A−B

$$A-B = \{x \mid x \in A \text{ and } x \notin B\}$$

for example, if two sets A and B are given as

$A = \{a, c, d, g, h, i\}$ $B = \{a, b, c, e, f, g, i\}$

Then

$A-B = \{d, h\}$

and

$B-A = \{b, e, f\}$

Cartesian Product:- If A and B are the two sets then the cartesian product of A and B denoted as A×B

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B, \forall a, \& \forall b\}$$

Concatenation:- If A and B are the two sets then the concatenation of A and B denoted as A·

$$A \cdot B = \{ab \mid a \text{ is in } A \text{ and } b \text{ is in } B\}$$

For example

if $A = \{0, 1\}$ and $B = \{1, 2\}$ then

$A \cdot B = \{01, 02, 11, 12\}$

Symmetric Difference:- The symmetric difference of two sets A and B (denoted by A△B) is defined as

$$A \triangle B = (A-B) \cup (B-A)$$

For example, If two sets A and B are given as

$A = \{1, 3, 5, 7, 9\}$ $B = \{1, 2, 4, 5, 6, 8, 9\}$

Then

$A-B = \{3, 7\}$

and

$B-A = \{2, 4, 6, 8\}$

Therefore $A \Delta B = \{3,7\} \cup \{2,4,6,8\}$

$= \{2,3,4,6,7,8\}$

③ **Relations :-** A relation from A to B is the subset of $A \times B$. Suppose R is the relation from A to B, then R is a set of ordered pair where each first element comes from A and each second element comes from B. That is for each pair $a \in A$ and $b \in B$, exactly one of the following is true.

(i) $(a,b) \in R$ (i.e, a is R-related to b, written as aRb)

(ii) $(a,b) \notin R$ (i.e, a is not R-related to b)

→ If $A = \{1,2,3\}$ (domain), and $B = \{a,b,c\}$ (range) and $R = \{(1,b),(2,c),(1,a),(3,a)\}$ is subset of $A \times B$, then R is a relation from A to B

Therefore,

$\{(1,b),(2,c),(1,a),(3,a)\} \in R$

can also be written as 1Rb, 2Rc, 1Ra, 3Ra. But $(2,b) \notin R$ Therefore 2 is not related to b.

(i) **Inverse of a Relation :-**

If R is a relation from a set A to set B, then the Inverse of R, denoted by R^{-1}, is the relation from B to A which consists of those ordered pairs which, when reversed belong to R.

for example:-

$R^{-1} = \{(b,a) / (a,b) \in R\}$

Let $A = \{a,b,c,d\}$, and $B = \{1,2,3,4\}$, let $R = \{(a,1),(a,3),(b,3),(c,4),(d,1)\}$ be a relation from A to B then

$R^{-1} = \{(1,a),(3,a),(3,b),(4,c),(1,d)\}$

Types of Relations and properties of Relations:-

If R is a relation on set S, it is said to be

(i) **Reflexive:-** A relation R on a set S is reflexive if aRa for every $a \in S$, i.e, if $(a,a) \in R$ for every $a \in S$.

(ii) **symmetric and anti-symmetric Relation:-**

If aRb implies bRa, for all a and $b \in S$ (symmetric so $a=b \Rightarrow b=a$)

(iii) **transitive Relation:-** A relation R on a set A is said to be transitive if whenever aRb and bRc, then aRc. it means, if whenever

$(a,b), (b,c) \in R$

Then

$(a,c) \in R$

R will not be said transitive if there exists $a,b,c \in A$ such that $(a,b)(b,c) \in R$ but $(a,c) \notin R$. As an example, if l_1, l_2 and l_3 are three lines such that l_1 is parallel to l_2 and l_2 is parallel to l_3, then l_1 is parallel to l_3, therefore the relation "is parallel to" is a transitive relation.

(iv) **Equivalance Relation:-**

If a relation is reflexive, transitive as well as symmetric it is said to be equivalance relation.

eg:- relation "=" on set of integers

(i) It is reflexive because $a=a$
$a \in I$

(ii) It is transitive because
if $a=b$ and $b=c$
imply $a=c$ where $a,b,c \in I$

(iii) It is symmetric because

If $a=b$

imply $b=a \Rightarrow a, b \in I$

Therefore, it is an equivalence relation.

④ **Strings :-** A string is a finite sequence of symbols and we use a letter [ω] to denote a string

Note :- ε-(epsilon) π-(Pie) con λ (lambda) con ∈ (belongs to)

Length of String :- The length of string is the number of letters in the string. suppose we define the function 'length' to compute the length of a string.

For example, if $a = xxxx$ is the string in the language L then length $(a) = 4$

Reverse of String :- Now we introduce a new function 'reverse'. If ω is a word in language L, then reverse (ω) is the same string of letters traversed backward. Called reverse of ω, even if this backward string is not a word in language L.

ex:- reverse$(xyz) = zyx$

→ **Concatenation :-** Basically concatenation is the operation applicable to strings to combine them. A particular string can be concatenated with any string and it self also.

example :-

{a, b} {a, b} = {aa, ab, ba, bb}

Prefix:- A prefix of a string is a substring of leading symbols of the string.

eg:- let $x = abc$

prefixes of x are ϵ, a, ab, abc

where, ϵ is an empty string with length zero

Proper prefix:-

Any prefix of a string other than the string itself is called as a proper prefix of that string.

example:-

If $w = abc$

Then ϵ, a and ab are the proper prefixes of w

Suffix:- A suffix of a string is any number of trailing symbols of the string

ex:- let $x = abc$

suffix of x are ϵ, c, bc, abc.

Proper suffix:-

Any suffix of a string other than the string itself is called as a proper suffix of that string.

example:-

If $w = abc$

Then $\epsilon, a, b, c, ab, bc$ and abc are the substring of the w. If $w = |n|$, then there exist $[n(n+1)/2 + 1]$ substrings of w

5→ **closure of relation:-** If P is a set of properties of relation R, then the P-closure of relation R is the smallest relation R' that includes all the pairs of R and possesses the properties in P.

① **Transitive closure of $R(R^{+})$:-**

It is defined as:

(i) If $(a,b) \in R$ then (a,b) is in R^+.
(ii) If $(a,b) \in R$ and $(b,c) \in R$ then (a,c) is in R^+

eg:- Let $S = \{1,2,3\}$ and
where R is a relation on S

$R = \{(1,2),(2,2),(2,3)\}$ where R is a relation on S

$R^+ = \{(1,2),(2,2),(2,3),(1,3)\}$

As $(1,2)$ and $(2,3)$ are the members, $(1,3)$ is added.

Reflexive and transitive closure of R (R^*):-

$$R^* = R^+ \cup \{(a,a) / a \in S\}$$

eg:- for the above set S and relation R

$R^* = \{(1,1)(1,2)(1,3)(2,2)(2,3)(3,3)\}$

$(1,1)$ and $(3,3)$ are added $(2,2)$ was already a member.

Partial ordering:- A relation R on a set S is said to be in partial order relation if R is
(i) a reflexive relation
(ii) an anti-symmetric relation
(iii) a transitive relation is called partial ordering

anti-symmetric means:- if aRb, it does not imply bRa

eg:- Find the transitive closure and the symmetric closure of the relation. $R = \{(1,2),(2,4),(4,5),(6,5)\}$

sol:- Transitive closure

$R^+ = \{(1,2),(2,4),(4,5),(6,5),(1,4),$ ~~[illegible]~~ $(1,5)\}$

reasoning = $(1,2)$ and $(2,4)$ are the members so $(1,4)$ is added
$(1,4)$ and $(4,5)$ are members $\Rightarrow (1,5)$ is added

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Cross-out

symmetric closure:-

$R^{*} = \{(1,2),(2,4),(4,5),(6,5),(2,1),(4,2),(5,4),(5,6)\}$

reasoning:-

(1,2) is member therefore (2,1) is added

similarly (4,2) (5,4) and (5,6) are added.

5 Diagonalization:-

If R is a binary relation on some set A then R can be represented by a table, with rows and columns labled with elements of A, and 1 in a box with row index a, and column index b indicates that (a,b) is in R, where as 0 in a box with row index a, and column index b indicates that (a,b) is not in R, such a representation is shown below.

R	a	b	c	d
a	1	0	1	0
b	0	0	1	1
c	0	1	1	0
d	1	1	1	0

If we define diagonal set D of R as follows

D = {a | a is in A, and (a,a) = 0}

binary relation:- we see that if R is the relation of A to B, then $R \subseteq A \times B$. In particular, if any subset $A \times A$ defining a relation in A is called a binary relation

(6) PIGEON HOLE PRINCIPLE:-

If A and B are two nonempty finite sets and $|A| > |B|$ then there is no one-to-one function from A to B. If we attempt to pair off the elements of A (the pigeons) with elements of B (the pigeon holes) sooner or later we will have to put more than one pigeon in a pigeon hole.

Suppose a postman distributes 51 letters in 50 mailboxes (pigeonholes). Then it is evident that some mailbox will contain at least two letters. This is enunciated as a mathematical principle called the pigeonhole principle.

If n objects are distributed over m places and $n > m$, then some place receives at least two objects.

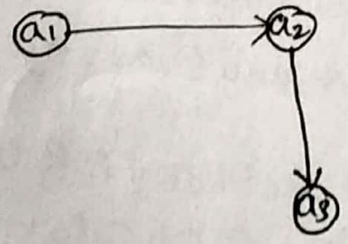
In other words. If we attempt to pair off the element of S_1 with element of S_2, sooner or later we will have to put more than one pigeon in a pigeonhole.

For example

if $S_1 = \{a_1, a_2, a_3\}$ then $|S_1| = 3$

if $S_2 = \{(a_1, a_2), (a_2, a_3)\}$ Then $|S_2| = 2$

This would only holds for an undirected graph that is no cycles



④ **concept of language:-** A certain specified set of strings of characters from the alphabet is called the language.

collection of strings from alphabet Σ is called language.

Let $\Sigma = \{x\}$

→ ϕ (null set) and the set consisting of empty string $\{\epsilon\}$ are also languages

→ we can define a language by saying that any non-empty string of alphabet characters is a word.

$$L_1 = \{x, xx, xxx, \ldots\}$$

we can also write

$$L_1 = \{x^n \text{ for } n = 1,2,3 \ldots\}$$

→ set of all strings over a fixed alphabet Σ is language denoted by Σ^*

ex: let $\Sigma = \{a\}$ then

$$\Sigma^* = \{\epsilon, a, aa, aaa, \ldots\}$$

let $\Sigma = \{0,1\}$ then

$$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$$

operations on languages:-

(i) concatenation:-

the concatenation $L_1 L_2$ of languages L_1 and L_2 is defined by

$$L_1 L_2 = \{uv / u \in L_1, v \in L_2\}$$

eg:- let $\Sigma = \{a,b\}$ and L_1 and L_2 are languages over Σ

ans $L_1 = \{ab, bb\}$ $L_2 = \{a, b\}$

$$L_1 L_2 = \{aba, abb, bba, bbb\}$$

$$L_2 L_1 = \{aab, abb, bab, bbb\}$$

example:- (normal language example)

$\Sigma = \{a,b\}$

$L_1 = \{ab, ba, aa, bb, \epsilon\}$

$L_2 = \{w \mid w = (ab)^k, k = 0,1,2,3 \dots\}$

$= \{\epsilon, ab, abab, ababab \dots\}$

example:-

$L = \{a^n b^n \mid n \geq 0\}$

~~L·L=~~

$L \cdot L = L^2 = \{a^n b^n a^m b^m \mid n \geq 0, m \geq 0\}$

The string aabb aaabbb is in L^2. The star-closure or kleene closure of a language is defined as

$$L^* = L^0 \cup L^1 \cup L^2 \dots$$

$$= \bigcup_{i=0}^{\infty} L^i$$

Positive closure as

$$L^+ = L^1 \cup L^2 \dots$$

$$= \bigcup_{i=1}^{\infty} L^i$$

~~Chomsky hierarchy:-~~

However, there may be some other languages that appear when we study automata. Some examples are as follows

(a) The language of all strings consisting of n 0's followed by n 1's, for some $n \geq 0$

Ans $= \{\epsilon, 01, 0011, 000111, \dots\}$

(b) the set of strings over 0's and 1's with equal number of each:

$= \{\epsilon, 01, 10, 0011, 0101, 1001, \dots\}$

(c) The set of binary numbers whose value is a prime:

{10, 11, 101, 111, 1011, ---}

(d) Σ^* is a language for any alphabet Σ

(e) ϕ The empty language, is a language over any alphabet

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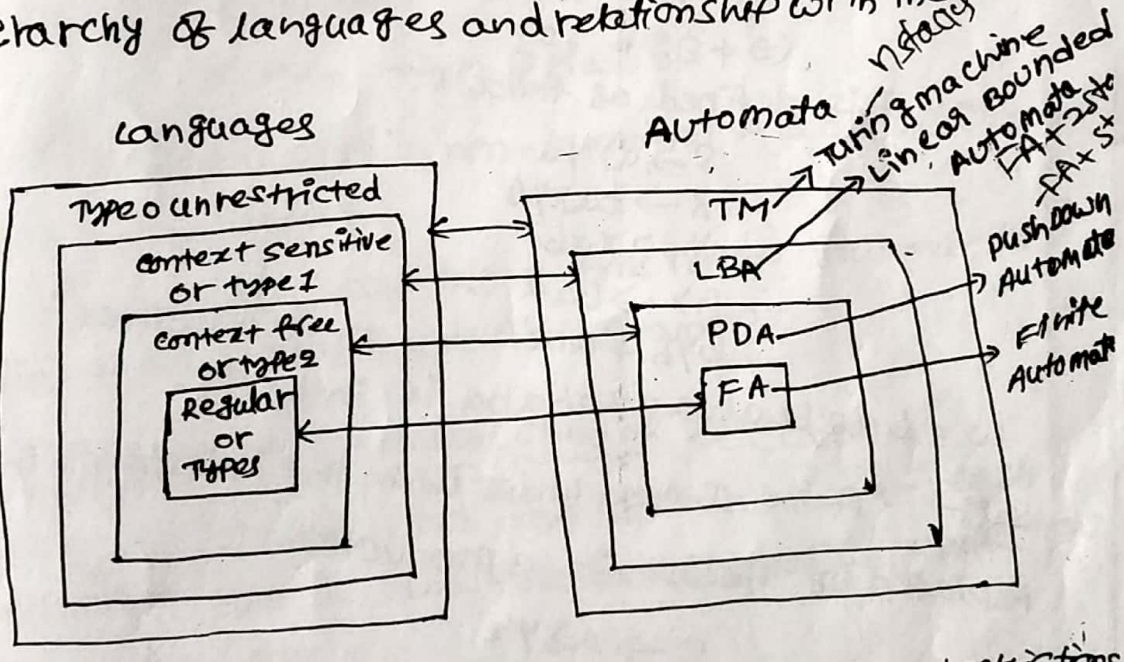
(7) **Chomsky HIERarchy:-** The four classes of languages that we have studied regular, context free, context sensitive and recursively enumerable are often reffered to as the chomsky hierarchy. chomsky classified the grammar into four types in terms of productions as type3, type2, type1 and type 0. each level of hierarchy can be characterized by a class of grammars and by a certain type of abstract machine or model of computation. The following picture shows hierarchy of languages and relationship with the machines



Type 0: (UNRESTRICTED grammar) :- There are no restrictions on the productions rules of this class. This class generates the largest family of grammars permitting productions of the form

$$\alpha \to \beta$$

where α and β are arbitary strings of symbols with $\alpha \neq \epsilon$

such grammars are known as unrestricted grammars. Every type 0 grammar generates a recursively enumerable set. It is called recursively enumeraable language. A turing Machine is constructed to recognize the sentences generated by this grammar

The unrestricted grammar is defined as

$$G = (V_n, V_t, P, S)$$

where V_n = a finite set of non-terminals
V_t = a finite set of terminals
S = starting non-terminal, $S \in V_n$
and P is set of productions of the fallowing form

$$\alpha \to \beta$$

<u>example:-</u>

let $G = (V_n, V_t, P, S)$
where
$V_n = \{S, X, Y\}$
$V_t = \{a, b\}$
$S = \{S\}$

and P is defined as fallows:

$S \to aXYa$
$X \to baXYb$
$Y \to Xab$
$aX \to baa$
$bYb \to abab$ Test whether

$w = baabbabaaabbaba$ is in $L(G)$.

<u>Sol:-</u> Here I am underline the substring to be replaced by the use of a production:

$S \to a\underline{X}Ya$
$\to baa\underline{Y}a$
$\to baa\underline{X}aba$
$\to baa\underline{baXY}baba$
$\to baabbaa\underline{Y}baba$
$\to baabbaa\underline{X}abbaba$
$\to baabbabaaabbaba$
$\to w$

so $w \in L(G)$.

CONTEXT- SENSITIVE GRAMMAR: TYPE 1 :-

In this class, the restriction on each production rule $\alpha \to \beta$ is that the length of the consequent β is at least as much as the antecedent α, except for $S \to \epsilon$.

*** also the start symbol 'S' does not apper on the right hand side of any production.

This grammar is also called a context sensitive grammar (CSG), and the language it generates is known as the context sensitive language (CSL) or type 1 language. The term context sensitive originates from productions of the form

$$\alpha_1 A \alpha_2 \to \alpha_1 \beta \alpha_2 \quad (\beta \neq \epsilon)$$

*** Replacement of a non-terminal A is allowed by β only in the context

*** so we can say that ϵ free type-0 grammar is said to be length increasing grammar.

Context-Free grammar (CFG) OR type 2 :-

In this class, the left hand side of each production rule is non-terminal symbol i.e, productions are of the form

$$A \to \alpha$$

where A is a non-terminal

$$\alpha \in (V \cup T)^*$$

The productions of the form $A \to \epsilon$ (beside $A = S$) are also permitted. In addition, S is allowed to appear on right hand side of a production.

This grammar is referred to as CFG

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The grammars of most PL's approximate to this grammar.

Regular grammar (or) type 3:

For this class in each production rule the left hand side is a non-terminal symbol and the right hand side contains at most one non-terminal symbol which is the right most (or left most) symbols. eg:

① $A \to \alpha B$, $A \to \alpha$

AB - nonterminal
α - terminal

This type of grammar is also called regular grammar.

This grammar is too primitive for PL's.

⑧ Induction and proof methods:

In practice, inductions is used to prove assertions of the form "for all natural numbers n, property P is true" in following way

1) In the basic step we show that 0 belongs to A. That is P is true for zero, this is called as basic step

2) Then we make an assumption that for some fixed but arbitary $n \geq 0$, P is true for each natural number 0, 1, --- n. This is called induction hypothesis

3) Then using induction hypothesis that P for all natural number up to and including n for some arbitary n, we prove that P is true for n+1. This we call as induction step

example:-

1. Show for any $n \geq 0$

$$1+2+ \cdots +n = (n^2+n)/2$$

Basic step:-

for $n=0$

LHS $=0$

RHS $=(0^2+0)/2$

$=0$

L.H.S = R.H.S

Induction hypothesis:-

Assume that for some $n \geq 0$

$1+2+ \cdots +n = (n^2+n)/2$ for every natural number up to and including n.

Induction step:-

for $n+1$, the L.H.S of given equation is:

$1+2+ \cdots +(n+1)$

$= (1+2+3+ \cdots +n)+(n+1)$

$= (n^2+n)/2+(n+1)$

$= (n^2+n+2n+2)/2$

$= (n^2+3n+2)/2$

$= ((n^2+2n+1)+(n+1))/2$

$= ((n+1)^2+(n+1))/2$

$=$ RHS

which we want to be the R.H.S for $n+1$

example 2 :-

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + n\cdot(n+1)(n+2) = \frac{\{n\cdot(n+1)\cdot(n+2)\cdot(n+3)\}}{4}$$

for any $n \geq 0$

Ans Basic step :

for $n = 0$

LHS = 0

$$RHS = \frac{0\cdot(0+1)\cdot(0+2)\cdot(0+3)}{4} = \frac{0\cdot1\cdot2\cdot3}{4} = 0$$

LHS = RHS

Induction hypothesis :

Assume that for every natural number up to and including n for some arbitary $n \geq 0$

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + n\cdot(n+1)\cdot(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Induction step :-

for $n+1$ the L.H.S of given equality is

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots + n(n+1)(n+2) + (n+1)(n+2)(n+3).$$

By induction hypathesis.

$$= \frac{n(n+1)(n+2)(n+3)}{4} + (n+1)(n+2)(n+3)$$

$$= \frac{n(n+1)(n+2)(n+3) + 4(n+1)(n+2)(n+3)}{4}$$

$$= \frac{(n+1)(n+2)(n+3)(n+4)}{4}.$$

= as was to be shown R.H.S

Formal Grammar:- A formal grammar is a notation used to specify a language and is made of four components - nonterminals or variables, terminals production rules and a start symbol

Therefore, a formal grammar is a 4-tuple

(V, T, P, S) where

V - is a finite set of symbols called as 'variables' or "non-terminals".

T - is a finite set of symbols called as "terminals".

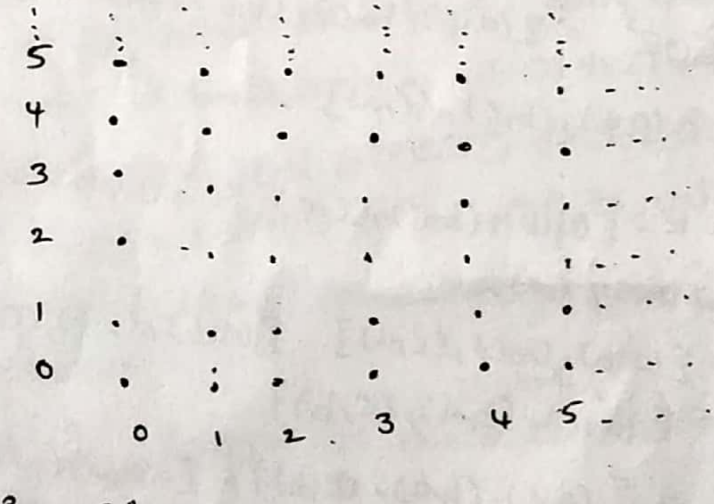
P - is a finite set of rules called as "production rules".

S -> is a member of V called as "start-symbol".

Countability

Proof of countability

The cartesian product $N \times N$ is countable.



Technique:- construct a one-to-one, onto function $f: N \times N \rightarrow N$

more generally, The cartesian product of a finite number of countable sets is countable

countable and uncountable

A set that has the same cardinality as the natural numbers

$N = \{0, 1, 2, \ldots\}$ is denumerable

Every infinite subset of N is denumerable

x is countable if it is finite or denumerable

y is uncountable if it not countable

ex:- The set of real numbers is uncountable

CR

eg:- Find R^+, if $R = \{(a,b), (b,c), (b,d)\}$ be a relation in $\{a, b, c, d\}$

sol:- $R = \{(a,b), (b,c), (b,d)\}$

$R^2 = R \cdot R = \{(a,b), (b,c), (b,d)\} \cdot \{(a,b), (b,c), (b,d)\}$

$= \{(a,c), (a,d)\}$

$R^3 = R^2 \cdot R = \{(a,c), (a,d)\} \cdot \{(a,b), (b,c), (b,d)\}$

$= \{\phi\}$

No pair

$R^4 = R^5 = \phi$

$R^+ = R \cup R^2 \cup R^3 = \{(a,b), (b,c), (b,d), (a,c), (a,d)\}$

eg:- R^+ if $R = \{(a,b), (b,c), (c,a)\}$ find R^+

eg:- find R^* if $R = \{(a,b), (b,c), (c,a)\}$ is a relation in $\{a, b, c\}$

$R = \{(a,b), (b,c), (c,a)\}$

$R^2 = R \cdot R = \{(a,b), (b,c), (c,a)\} \cdot \{(a,b), (b,c), (c,a)\}$

$= \{(a,c), (b,a), (c,b)\}$

$R^3 = R^2 \cdot R = \{(a,c), (b,a), (c,b)\} \cdot \{(a,b), (b,c), (c,a)\}$

$= \{(a,a), (b,b), (c,c)\}$

$R^4 = R^3 \cdot R = \{(a,a), (b,b), (c,c)\} \cdot \{(a,b), (b,c), (c,a)\}$

$= \{(a,b), (b,c), (c,a)\} = R$

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$R^5 = R^4 . R = R.R = R^2$

$R^6 = R^5 . R = R^2 . R = R^3$

$R^7 = R^6 . R = R^3 . R = R^4 = R.$

Then any R^n is one of R, R^2 or R^3 Hence

$R^+ = R \cup R^2 \cup R^3$

$= \{(a,b),(b,c),(c,a)\} \cup \{(a,c),(b,a),(c,b)\} \cup \{(a,a),(b,b),(c,c)\}$

Hence $R^* = R^+ \cup R$

$= \{(a,b),(b,c),(c,a)\} \cup \{(a,c),(b,a),(c,b)\} \cup \{(a,a),(b,b),$
$(c,c)\} \cup \{(a,a),(b,b),(c,c)\}$

$= \{(a,b),(b,c),(c,a),(b,a),(c,b),(a,a),(b,b),(c,c$

~~$1^2+2^2+3^2+ \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$~~

① **countability:-** Infinite set that can be placed in one to one correspondence with the set of natural number (N) are said to be countably infinite.

→ finite set are always countable.

→ some Infinite sets are uncountable e.g set of real number.

→ A set is said to be countable if it is finite or countable infinite.

② **Decidability:-** In computability theory a set of natural number is called recursive, computable or decidable

→ If there is an algorithm which terminate after a finite amount of time and correctly decides whether or not a given number belongs to the set is called decidability

→ A set which is not computable is called non computable or undecidable.

③ **Symbol:-** Symbol is an user defined or an abstract entity.

eg:- Letters a, b, c, - - - - z

Digits 0, 1, 2, - - - 9

special character +, -, \, *, <, >, = etc, are symbols.

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④ **Alphabet (Σ)**:- It is a finite set of symbols, It is denoted by Σ or capital letters like A, B, C.

Eg:- $\Sigma = A = \{a, b, c, \ldots z\}$

$B = \{0, 1, 2, \ldots 9\}$

$C = \{+, -, *, <, >\}$

⑤ **Sub sequences**:- It is the string formed by deleting zero or more not necessary contiguous symbol from string s.

E.g:- ale is a sub sequence of apple

⑥ **Sub strings**:- A substring is obtained by deleting a prefix and a suffix from a string S. Hence every prefix and every suffix of string S is a substring of S. Substring also includes string itself & ?

Pigeon Hole principle:-

eg: Find the minimum number of students in a class so that three of them are born in same month.

sol:- Here $n = 12$ (months) which represents the number of pigeon holes

$k+1 = 3$ or $k = 2$

Hence the minimum number of pigeons

$= kn+1$

$= 25$ students.