SUB :- EMF

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UNIT - 1. ELECTROSTATICS :-

Introduction to Cartesian, Cylindrical and Spherical co-ordinate System, Electric Field Intensity, Flux density, Gauss's Divergence theorem, Electric Potential and Potential Gradient.

UNIT - 2. MAGNETOSTATICS :-

Current density and continuity equation, Biot-Saver's law, Ampere's circuital law & applications, Magnetic flux & flux density, Scaler & Vector Magnetic Potentials.

UNIT - 3. MAXWELLS EQUATIONS & BOUNDARY CONDITIONS :-

Maxwell's equations for steady fields, Maxwell's equations for time varying fields, Electric & Magnetic boundary conditions.

UNIT - 4 ELECTROMAGNETIC WAVES :-

Electromagnetic wave equation, wave propagation in free space, in a perfect dielectric, & perfect conductor Skin effect, Poynting vector and Poynting theorem, reflection and refraction of uniform plane wave at normal incidence plane, reflection at oblique incident angle.

UNIT - 5 WAVEGUIDES :-

Introduction, wave equation in Cartesian Co-ordinate Rectangular wave guide, TE, TM, TEM waves in Rectangu-lar guides, wave impedance, losses in wave guide, Introduction to circular waveguide.

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UNIT - 6 :- RADIATIONS :-

Retarded potential, Electric & Magnetic fields due to oscillating dipole (alternating current element), Power radiated and radiation resistance, application to short monopole and dipole, Antenna efficiency, Beam-width, Radiation Intensity, Directive Gain Power Gain & Front to Back Ratio, Advance topics on the subject.

Reference Book
A.U. Tinguriya - EMT

UNIT - 1

ELECTROSTATICS

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Rectangular / Cartesion Co-ordinates System :-

A (1, 2, 3)    Position Vectors.

$1\hat{a_x} + 2\hat{a_y} + 3\hat{a_z}$

$x \quad y \quad z$

$\hat{a_x} \quad \hat{a_y} \quad \hat{a_z}$

B (4, 5, 6)

$4\hat{a_x} + 5\hat{a_y} + 6\hat{a_z}$

$\vec{AB} = \underset{\text{source}}{A} \longrightarrow \underset{\text{destination}}{B}$

= Destination - Source

= B - A

* In dot product we get,

$\hat{a_x} \cdot \hat{a_x} = 1 \quad , \quad \hat{a_x} \cdot \hat{a_y} = 0 \quad , \quad \& \quad \hat{a_x} \cdot \hat{a_z} = 0 \, .$

$A \cdot B = 1\hat{a_x} \cdot 4\hat{a_x} + 2\hat{a_y} \cdot 5\hat{a_y} + 3\hat{a_z} \cdot 6\hat{a_z}$

$\therefore A \cdot B = 4 + 10 + 18 = 32$ It is scaler quantity

* Cross product we get

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$= \hat{a_x}(12-15) - \hat{a_y}(6-12) + \hat{a_z}(5-8)$

$= -3\hat{a_x} + 6\hat{a_y} - 3\hat{a_z}$

$\therefore \quad C = (3, 4, 2)$

find vector $\hat{a_A}$, $\hat{a_B}$, $\hat{a_{CA}}$

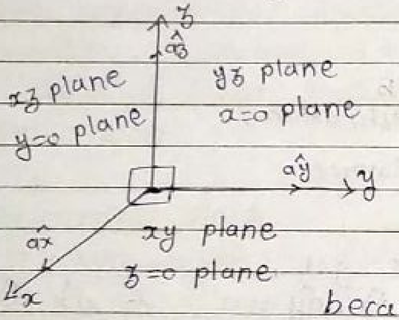
$$\hat{a_A} = \frac{\vec{A}}{|A|} = \frac{1\hat{a_x} + 2\hat{a_y} + 3\hat{a_z}}{\sqrt{1^2+2^2+3^2}} = \frac{1\hat{a_x} + 2\hat{a_y} + 3\hat{a_z}}{\sqrt{14}}$$

$$\hat{a_B} = \frac{\vec{B}}{|B|} = \frac{4\hat{a_x} + 5\hat{a_y} + 6\hat{a_z}}{\sqrt{4^2+5^2+6^2}} = \frac{4\hat{a_x} + 5\hat{a_y} + 6\hat{a_z}}{\sqrt{16+25+36}} = \frac{-\text{"}-}{\sqrt{77}}$$

$$\hat{a_{CA}} = \frac{\vec{CA}}{|CA|} = \frac{C-A}{|CA|} = \frac{3\hat{a_x} + 4\hat{a_y} + 2\hat{a_z} - (1\hat{a_x} + 2\hat{a_y} + 3\hat{a_z})}{|CA|}$$

$$\hat{a}_{CA} = \frac{2\hat{a}_x + 2\hat{a}_y - \hat{a}_z}{|CA|} = \frac{2\hat{a}_x + 2\hat{a}_y - \hat{a}_z}{\sqrt{2^2+2^2+1^2}} = \frac{2\hat{a}_x + 2\hat{a}_y - \hat{a}_z}{3}$$

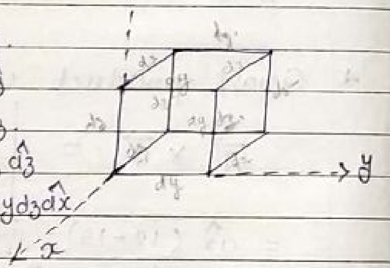
## * Cartesion / Rectangular System *




All the co-ordinates of the cartesion system mutually $\perp^r$ to each other & their unit vectors are also mutually $\perp^r$ to each other because they are in same direction to the co-ordinates x, y, z.

MMP

| | | | |
|---|---|---|---|
| i) Co-ordinates | $x$ | $y$ | $z$. |
| ii) unit vectors | $\hat{a}_x$ | $\hat{a}_y$ | $\hat{a}_z$ |
| iii) differential | $dx$ | $dy$ | $dz$. |
| length dL | $dx\hat{a}_x$ | $dy\hat{a}_y$ | $dz\hat{a}_z$ |
| iv) Differential Source ds | $dx\,dy\,\hat{a}_z$ | $dx\,dz\,\hat{a}_y$ | $dy\,dz\,\hat{a}_x$ |
| v) Differential Volume dv | | $dx\,dy\,dz$. | |



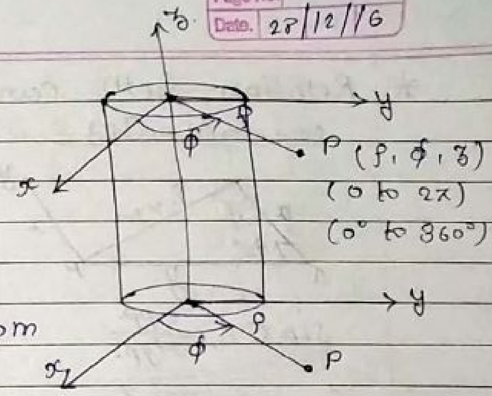

## (2) Cylindrical Co-ordinate Systems :-

| | | | |
|---|---|---|---|
| i) Co-ordinates | $\rho$ | $\phi$ | $z$. |
| ii) Unit Vectors | $\hat{a}_\rho$ | $\hat{a}_\phi$ | $\hat{a}_z$ |
| iii) differential | $d\rho$ | $\rho\,d\phi$ | $dz$. |
| length dl | $d\rho\,\hat{a}_\rho$ | $\rho\,d\phi\,\hat{a}_\phi$ | $dz\,\hat{a}_z$ |

--- ($\because$ length of arc = $\rho . d\phi$ i.e. side x angle)



| | |
|---|---|
| iv) differential Surface ds | $\rho.d\rho.d\phi\,\hat{a}_z$<br>$\rho.d\phi\,dz\,\hat{a}_\rho$<br>$d\rho\,dz\,\hat{a}_\phi$ |
| v) differential Volume dv | $\rho.d\rho.d\phi.dz$. |




$\rho$ = distance of point P from +ve z-axis.

$\phi$ = angle betn $\rho$ & +ve x-axis (0 to $2\pi$ in radius) (0° to 360° in degree)

z = distance of point P from origine in +ve z directn.

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## (3) Spherical Co-ordinate System :- $(r, \theta, \phi)$

MMP

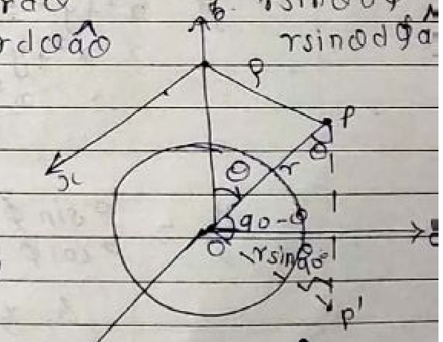
| | | | |
|---|---|---|---|
| i) Co-ordinates. | $r$ | $\theta$ | $\phi$ |
| ii) Vector units | $\hat{a}_r$ | $\hat{a}_\theta$ | $\hat{a}_\phi$ |
| iii) Differential | $dr$ | $r\,d\theta$ | $r\sin\theta\,d\phi$ |
| length dL | $dr\,\hat{a}_r$ | $r\,d\theta\,\hat{a}_\theta$ | $r\sin\theta\,d\phi\,\hat{a}_\phi$ |

iv) Differential Surface ds. $r\,dr\,d\theta\,\hat{a}_\phi$

$r\,dr\sin\theta\,d\phi\,\hat{a}_\theta$

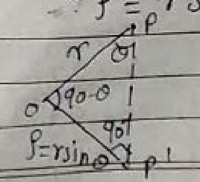
$r^2 d\theta\,\sin\theta\,d\phi\,\hat{a}_r$

v) differential volume dv. $r^2.dr.d\theta.\sin\theta.d\phi$




$\sin\theta = \frac{\rho}{r}$

$\therefore \rho = r\sin\theta$.




The range of $\theta$ is $0 - \pi$ or $0° - 180°$. & it is the angle betn the r & z-axis.

$\phi$ is the angle betn the $r\sin\theta$ and x-axis.

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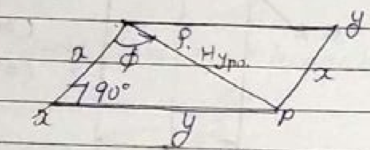
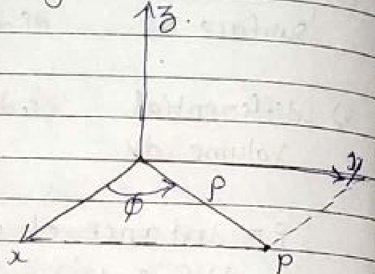
* Relation betn cartesion & cylindrical co-ordinates *




$\therefore \sin\theta = \frac{\text{opp.}}{\text{Hyp.}}$

$\cos\theta = \text{adj}/\text{Hypo.}$

$\therefore \cos\phi = \frac{x}{\rho} \Rightarrow \boxed{x = \rho\cos\phi}$

$\sin\phi = \frac{y}{\rho} \Rightarrow \boxed{y = \rho\sin\phi}$

$z = z.$

These is the relation with respect to x, y, & z.

* Now, in terms of ρ, φ & z. the relationi

$x^2+y^2 = \rho^2\cos\phi^2 + \rho^2\sin\phi^2$

$\therefore x^2+y^2 = \rho^2(\sin^2\phi + \cos^2\phi)$

$\therefore x^2+y^2 = \rho^2$

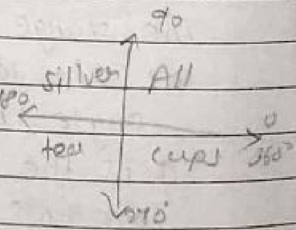
$\therefore \boxed{\sqrt{x^2+y^2} = \rho}$

$\frac{y}{x} = \frac{\rho\sin\phi}{\rho\cos\phi}$ $\therefore \frac{y}{x} = \tan\phi$ $\therefore \boxed{\phi = \tan^{-1}(y/x)}$

& $z = z$.

* Dot product conversion:

| • | $\hat{a\rho}$ | $\hat{a\phi}$ | $\hat{az}$ |
|---|---|---|---|
| $\hat{ax}$ | $\cos\phi$ | $\cos(90+\phi)$ $-\sin\phi$ | 0 |
| $\hat{ay}$ | $\sin\phi$ | $\sin(90+\phi)$ $\cos\phi$ | 0 |
| $\hat{az}$ | 0 | 0 | 1 |

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* Relation betn r, θ & φ bel with respect to x, y & z.

$\therefore$ side opposite to angle φ

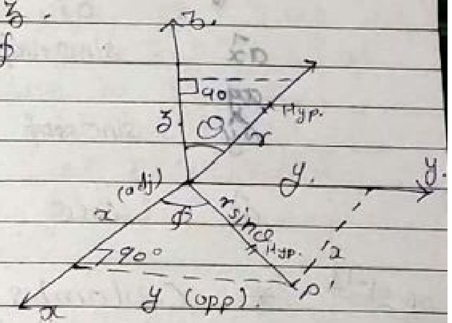
$\sin\phi = \frac{y}{r\sin\theta}$

$\therefore \boxed{y = r\sin\theta\sin\phi}$

side adjacent to angle φ

$\cos\phi = \frac{x}{r\sin\theta}$.

$\therefore \boxed{x = r\sin\theta\cos\phi}$




side adjacent to angle θ

$\cos\theta = \frac{z}{r}$ $\therefore \boxed{z = r\cos\theta}$

For r, θ, & φ.

$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$\therefore x^2+y^2+z^2 = r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$\therefore x^2+y^2+z^2 = r^2\sin^2\theta + r^2\cos^2\theta$

$\therefore x^2+y^2+z^2 = r^2(\cos^2\theta + \sin^2\theta)$.

$\therefore \boxed{r = \sqrt{x^2+y^2+z^2}}$

eqn of z implies:-

$z = r\cos\theta$.

$\therefore \cos\theta = z/r$

$\therefore \boxed{\theta = \cos^{-1}(z/r)}$ OR.

$\boxed{\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)}$

$\therefore$ divide eqn y by eqn x., we get

$\frac{y}{x} = \frac{\cancel{r\sin\theta}\sin\phi}{\cancel{r\sin\theta}\cos\phi} = \tan\phi$.

$\therefore \boxed{\phi = \tan^{-1}(y/x)}$

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## * Dot Product Conversion *

| $\cdot$ | $\hat{a_r}$ | $\hat{a_\theta}$ | $\hat{a_\phi}$ |
|---|---|---|---|
| $\hat{a_x}$ | $\sin\theta\cos\phi$ | $\sin(90+\theta)\cos\phi$ <br> $\cos\theta\cos\phi$ | $-\sin\phi$ |
| $\hat{a_y}$ | $\sin\theta\sin\phi$ | $\sin(90+\theta)\sin\phi$ <br> $\cos\theta\sin\phi$ | $\cos\phi$ |
| $\hat{a_z}$ | $\cos\theta$ | $\cos(90+\theta)$ <br> $-\sin\theta$ | $0$ |

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## * Coulombs Law *

States that, "The force extended is directly proportional to the product of the charges and inversly proportional to the square of the distance betn them.

i.e. $F \propto \frac{Q_1 Q_2}{r^2}$

$\therefore F = k \frac{Q_1 Q_2}{r^2}$

$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$ --- $(\because k = \frac{1}{4\pi\epsilon_0})$

$\boxed{\therefore \vec{F} = \frac{Q_1 Q_2}{9\times10^9 r^2}\hat{a_r}}$ N --- $(\because k = 9\times10^9)$

$\therefore F = \frac{Q_s Q_d}{4\pi\epsilon_0 r^2}\hat{a_r}$

$\frac{F}{Q_d} = \frac{Q_s}{4\pi\epsilon_0 r^2}\hat{a_r}$

Imp. $\boxed{E = \frac{Q_s}{4\pi\epsilon_0 r^2}\hat{a_r}}$ V/m.

where, E = Electric field Intensity.

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## * NUMERICALS *

1) Calculate 'E' at M (3, -4, 2) in free space caused by A) point - A charge $Q_1 = 2\mu C$ at $P_1$ (0, 0, 0). B) A charge $Q_2 = 3\mu C$ at $P_2$ (-1, 2, 3) c) Both due to $Q_1$ & $Q_2$.

Soln:- Given :- $Q_1 = 2\mu C$ at $P_1(0,0,0)$

$Q_2 = 3\mu C$. at $P_2(-1,2,3)$

Both ($Q_1$ & $Q_2$).

Find - E at M (3, -4, 2).

a) $E = \frac{Q_s}{4\pi\epsilon_0 |r|^2}\hat{a_r}$

$Q_s = 2\mu C = 2\times10^{-6}$ C.

$\frac{1}{4\pi\epsilon_0} = 9\times10^9$

$\therefore \vec{r} = d - s$

$= (3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z}) - 0$

$= 3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z}$

$|r| = \sqrt{3^2+(-4)^2+(2)^2} = \sqrt{9+16+4} = \sqrt{29}$

$\hat{a_r} = \frac{\vec{r}}{|r|} = \frac{3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z}}{\sqrt{29}}$

$\therefore E = \frac{2\times10^{-6}\times9\times10^9}{(\sqrt{29})^2\times\sqrt{29}}(3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z})$

$\therefore E = \frac{18\times10^3(3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z})}{29\sqrt{29}}$

$\therefore E = \cancel{54\times10^3\hat{a_x} - 40}$ $115.25(3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z})$

$\therefore \boxed{E = 345.75\,\hat{a_x} - 461\,\hat{a_y} + 230.50\,\hat{a_z}}$ V/m

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b) $E = \dfrac{Qs}{4\pi\epsilon_d |r^2|} \hat{a_r}$

$\therefore Q_s = 3\mu C = 3\times10^{-6}\,C.$

$\dfrac{1}{4\pi\epsilon_0} = 9\times10^{9}$

$\vec{r} = d - s$

$= (3\hat{a_x} - 4\hat{a_y} + 2\hat{a_z}) - (-\hat{a_x} + 2\hat{a_y} + 3\hat{a_z})$

$\vec{r} = 4\hat{a_x} - 6\hat{a_y} - \hat{a_z}$

$\therefore |r| = \sqrt{4^2 + (-6)^2 + (-1)^2} = \sqrt{16+36+1} = \sqrt{53}$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{4\hat{a_x} - 6\hat{a_y} - \hat{a_z}}{\sqrt{53}}$

$\therefore E = \dfrac{3\times10^{-6} \times 9\times10^{9}(4\hat{a_x} - 6\hat{a_y} - \hat{a_z})}{(\sqrt{53})^2 \times \sqrt{53}}$

$\therefore E = \dfrac{27\times10^{3}(4\hat{a_x} - 6\hat{a_y} - \hat{a_z})}{53\sqrt{53}}$

$\therefore E = 69.976\,(4\hat{a_x} - 6\hat{a_y} - \hat{a_z})$

$\therefore \boxed{E_{Q_2} = 279.90\,\hat{a_x} - 419.85\,\hat{a_y} - 69.976\,\hat{a_z}}$ V/m.

c) $E_T = E_{Q_1} + E_{Q_2}$

$= (345.75\,\hat{a_x} + 279.90\,\hat{a_x}) + (-461\,\hat{a_y} - 419.85\,\hat{a_y}) + (230.50\,\hat{a_z} - 69.976\,\hat{a_z})$

$\boxed{E_T = 625.65\,\hat{a_x} - 880.85\,\hat{a_y} + 160.52\,\hat{a_z}}$

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Q) A point charge $Q_1$ is located at $P(0,0,0)$ & $Q_2$ at $Q(0,0,1)$. Find $\mathcal{E}$ at $R(1,0,-1)$.

Soln:- Given :- $Q_1$ at $P(0,0,0) = 5\mu C$

$Q_2$ at $Q(0,0,1) = 10\mu C$.

Find $\mathcal{E}$ at $R(1,0,-1)$.

a) $\mathcal{E}$ due to $Q_1 = \dfrac{Qs}{4\pi\epsilon_0 |r^2|} \hat{a_r}$

$\vec{r} = d - s$

$= (\hat{a_x} + 0\hat{a_y} - \hat{a_z}) - 0$

$\vec{r} = \hat{a_x} + 0\hat{a_y} - \hat{a_z}$

$|r| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{a_x} - \hat{a_z}}{\sqrt{2}}$

$\therefore \mathcal{E}_{Q_1} = \dfrac{5\times10^{-6} \times 9\times10^{9}\,(\hat{a_x} - \hat{a_z})}{(\sqrt{2})^2 \cdot \sqrt{2}}$

$\therefore \mathcal{E}_{Q_1} = \dfrac{45\times10^{3}\,(\hat{a_x} - \hat{a_z})}{2\sqrt{2}}$

$\therefore \mathcal{E}_{Q_1} = 15909.90\,(\hat{a_x} - \hat{a_z})$

$\therefore \mathcal{E}_{Q_1} = 15909.90\,\hat{a_x} - 15909.90\,\hat{a_z}$ v/m.

b) $\mathcal{E}$ due to $Q_2 = \dfrac{Qs}{4\pi\epsilon_0 |r^2|} \hat{a_r}$

$\therefore \vec{r} = (\hat{a_x} + 0\hat{a_y} - \hat{a_z}) - (0\hat{a_x} + 0\hat{a_y} + \hat{a_z})$

$\vec{r} = \hat{a_x} + 0\hat{a_y} + 2\hat{a_z}$.

$\therefore |r| = \sqrt{1^2 + 4} = \sqrt{5}$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{a_x} + 0\hat{a_y} + 2\hat{a_z}}{\sqrt{5}}$

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$\therefore \mathcal{E}a_2 = \dfrac{10\times10^{-6}\times9\times10^{9}}{5\times\sqrt5}(\hat{a_x} - 2\hat{a_z})$

$\therefore \mathcal{E}a_2 = \dfrac{90\times10^{3}(\hat{a_x}) - 2\hat{a_z}}{5\sqrt5}$

$\therefore \mathcal{E}a_2 = 8049.84\hat{a_x} - 16099.68\hat{a_z}$ V/m.

$\therefore \mathcal{E}a_T = \mathcal{E}a_1 + \mathcal{E}a_2$

$= (15909.90\,\hat{a_x} + 8049.84\hat{a_x}) + (0\hat{a_y} + 0\hat{a_y}) + (-15909.90\hat{a_z} + 0\hat{a_z})\,6099.68\,\hat{a_z})$

$\mathcal{E}a_T = \cancel{15909.9}\,\hat{a_x} + 0\hat{a_y} - \cancel{15909.90\,\hat{a_z}}$ V/m

$8049.84\,\hat{a_x}$

$\boxed{\mathcal{E}a_T = 23959.74\,\hat{a_x} - 32009.58\,\hat{a_z}}$ V/m.

3) A point charge $Q_A = 1\mu C$ is located at point $A(0,0,1)$ & another point charge $Q_B = -1\mu C$ is located at point $B(0,0,-1)$. Find $\mathcal{E}$ at $P(1,2,3)$.

Soln: Given: $Q_A = 1\mu C$ at $A(0,0,1)$

$Q_B = -1\mu C$ at $B(0,0,-1)$

$\mathcal{E}$ at $P(1,2,3)$.

A) $\mathcal{E}a_A$ due to $Q_A = \dfrac{Qs}{4\pi\epsilon_0|r^2|}\hat{a_r}$

$\therefore \vec{r} = (\hat{a_x} + 2\hat{a_y} + 3\hat{a_z}) - (0\hat{a_x} + 0\hat{a_y} + \hat{a_z})$

$\vec{r} = \hat{a_x} + 2\hat{a_y} + 2\hat{a_z}$

$\therefore |r| = \sqrt{1+4+4} = \sqrt9 = 3.$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{a_x} + 2\hat{a_y} + 2\hat{a_z}}{3} =$

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$\mathcal{E}a_A = \dfrac{1\times10^{-6}\times9\times10^{9}}{27}(\hat{a_x} + 2\hat{a_y} + 2\hat{a_z})$

$\therefore \mathcal{E}a_A = \dfrac{9\times10^{3}}{27}(\hat{a_x} + 2\hat{a_y} + 2\hat{a_z})$

$\therefore \mathcal{E}a_A = 333.33(\hat{a_x} + 2\hat{a_y} + 2\hat{a_z})$

$\therefore \boxed{\mathcal{E}a_A = 333.33\,\hat{a_x} + 666.66\,\hat{a_y} + 666.66\,\hat{a_z}}$ V/m

b) $\mathcal{E}a_B$ due to $Q_B = \dfrac{Qs}{4\pi\epsilon_0|r^2|}\hat{a_r}$

$\therefore \vec{r} = (a_x + 2\hat{a_y} + 3\hat{a_z}) - (0\hat{a_x} + 0\hat{a_y} - \hat{a_z})$

$\vec{r} = \hat{a_x} + 2\hat{a_y} + 4\hat{a_z}$

$|r| = \sqrt{1+4+16} = \sqrt{21}$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{\hat{a_x} + 2\hat{a_y} + 4\hat{a_z}}{\sqrt{21}}$

$\therefore \mathcal{E}a_B = \dfrac{-1\times10^{-6}\times9\times10^{3}(\hat{a_x} + 2\hat{a_y} + 4\hat{a_z})}{21\sqrt{21}}$

$\therefore \mathcal{E}a_B = \dfrac{-9\times10^{3}(\hat{a_x} + 2\hat{a_y} + 4\hat{a_z})}{21\sqrt{21}}$

$\therefore \mathcal{E}a_B = -93.52(\hat{a_x} + 2\hat{a_y} + 4\hat{a_z})$

$\therefore \boxed{\mathcal{E}a_B = -93.52\,\hat{a_x} - 187.04\,\hat{a_y} - 748.17\,\hat{a_z}}$ V/m

$\therefore \mathcal{E}_T = \mathcal{E}a_A + \mathcal{E}a_B$

$\boxed{\mathcal{E}_T = 239.81\,\hat{a_x} + 479.62\,\hat{a_y} - 81.51\,\hat{a_z}}$ V/m

$\epsilon_0 = 8.854 \times 10^{-12}$

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② $E = \dfrac{\rho_L}{2\pi\epsilon_0 \rho}\, \hat{a_\rho}$ V/m. where $\frac{1}{2\pi\epsilon_0} = 18 \times 10^9$

③ $E = \dfrac{\rho_s}{2\epsilon_0}\, \hat{a_N}$ V/m

④ Electric Flux Density $(D) = \epsilon_0 E$ Coulomb/$m^2$

for point charge $(D_Q) = \cancel{\epsilon_0} \times \dfrac{Q}{4\pi \cancel{\epsilon_0} r^2}\, \hat{a_r}$

$$\boxed{D_Q = \frac{Q}{4\pi r^2}\, \hat{a_r}} \quad C/m^2$$

for line charge $(D_{\rho_L}) = \cancel{\epsilon_0} \dfrac{\rho_L}{2\pi \cancel{\epsilon_0} \rho}\, \hat{a_\rho}$

$$\therefore \boxed{D_{\rho_L} = \frac{\rho_L}{2\pi\rho}\, \hat{a_\rho}} \quad C/m^2$$

for surface charge $(D_{\rho_s}) = \cancel{\epsilon_0} \dfrac{\rho_s}{2\cancel{\epsilon_0}}\, \hat{a_N}$

$$\therefore \boxed{D_{\rho_s} = \frac{\rho_s}{2}\, \hat{a_N}} \quad C/m^2$$

Note :- $D = \epsilon E$ where, $\epsilon = \epsilon_0 \epsilon_r$
and in free space $\epsilon_r = 1$
$\therefore D = \epsilon_0 E$.

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eqn for line charge

Electric field intensity due to point charge (Q) is

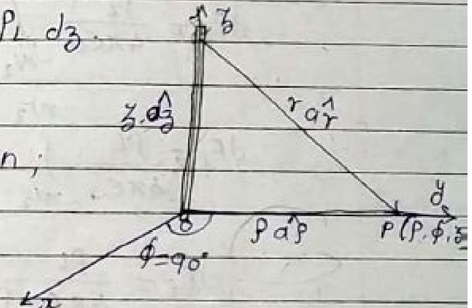
$$E = \frac{Q}{4\pi\epsilon_0 r^2}\, \hat{a_r}$$

consider the $\infty$ uniform line charge density $\rho_L$ c/m on z-axis.

$\therefore$ Differential electric field intensity $dE = \dfrac{dQ}{4\pi\epsilon_0 r^2}\, \hat{a_r}$

where, $dQ = \rho_l\, d_l = \rho_L\, dz$.

$\therefore dE = \dfrac{\rho_L\, dz}{4\pi\epsilon_0 r^2}\, \hat{a_r}$




$\therefore$ By vector law of addition,

$z\hat{a_z} + r\hat{a_r} - \rho\hat{a_\rho} = 0.$

$z\hat{a_z} + r\hat{a_r} = \rho\hat{a_\rho}$

$\therefore r\hat{a_r} = \rho\hat{a_\rho} - z.\hat{a_z}$

$\therefore |r| = \sqrt{\rho^2 + z^2}$

$\therefore \hat{a_r} = \dfrac{\vec{r}}{|r|} = \dfrac{\rho\hat{a_\rho} - z.\hat{a_z}}{\sqrt{\rho^2+z^2}}$

$\therefore dE = \dfrac{\rho_L\, dz\, (\rho\hat{a_\rho} - z.\hat{a_z})}{4\pi\epsilon_0 (\rho^2+z^2)\sqrt{\rho^2+z^2}}$

$\therefore dE = \dfrac{\rho_L\, dz\, (\rho\hat{a_\rho} - z.\hat{a_z})}{4\pi\epsilon_0 (\rho^2+z^2)(\rho^2+z^2)^{1/2}} = \dfrac{\rho_L dz(\rho\hat{a_\rho} - z\hat{a_z})}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}}$

Since the line charge is infinitely present on z-axis we get,

$dE = \dfrac{\rho_L\, dz\, (\rho\hat{a_\rho})}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}}$

$\therefore dE_\rho = \dfrac{\rho_L\, dz\, \rho}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}}$ $\qquad \therefore E = \int_{-\infty}^{\infty} dE$

$\therefore E = \int_{-\infty}^{\infty} \dfrac{\rho_L\, dz\, \rho}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}}$

$-\infty = \rho \tan\theta$ or $\infty = \rho\tan\theta$
$-\infty = \tan\theta$ $\quad \infty = \tan\theta$
$\theta = -\pi/2$ $\quad \therefore \theta = \pi/2$.

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$$dE_\rho = \frac{1}{4\pi\epsilon_0}\rho_L \int_{-\infty}^{\infty} \frac{\rho\, dz}{(\rho^2+z^2)^{3/2}}$$

$\therefore$ Put $z = \rho\tan\theta$ $\quad \therefore dz = \rho\sec^2\theta\, d\theta$

$$\therefore dE_\rho = \frac{1}{4\pi\epsilon_0}\rho_L \int_{-\pi/2}^{\pi/2} \frac{\rho\rho\sec^2\theta\, d\theta}{(\rho^2+\rho^2\tan^2\theta)^{3/2}}$$

$$\therefore dE_\rho = \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\rho^2\sec^2\theta\, d\theta}{(\rho^2\sec^2\theta)^{3/2}} \quad --- (\because 1+\tan^2\theta = \sec^2\theta)$$

$$dE_\rho = \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho^2}\sec^2\theta\, d\theta}{\rho^{\cancel{3}}\sec^{\cancel{3}}\theta}\, \hat{a}_\rho$$

$$\therefore dE_\rho = \frac{\rho_L}{4\pi\epsilon_0\rho}\int_{-\pi/2}^{\pi/2} \frac{d\theta}{\sec\theta}$$

$$\therefore dE_\rho = \frac{\rho_L}{4\pi\epsilon_0\rho}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta \quad -- (\because 1/\sec\theta = \cos\theta)$$

$$\therefore dE_\rho = \frac{\rho_L}{4\pi\epsilon_0\rho}(\sin\theta)_{-\pi/2}^{\pi/2}$$

$$\therefore dE_\rho = \frac{\rho_L}{4\pi\epsilon_0\rho}(1+1)$$

$$\therefore dE_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}$$

i.e. $\boxed{E = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho}$

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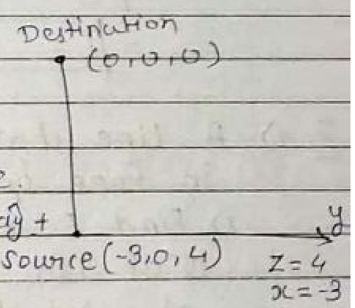
Q. A uniform line charge $\rho_L = 25\ nC/m$ lines on the $x = -3, z = 4$ in free space. Find E at i) origine ii) 2, 15, 3.

Soln:- Given :- $\rho_L = 25\ nC/m$
$\rho_L = 25\times10^{-9}\ C/m$

$$E = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho$$




$\therefore \frac{1}{2\pi\epsilon_0} = 18\times10^{9}$

$\therefore \vec{\rho}$ = Destination - Source

$\therefore \vec{\rho} = 0\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z - (-3\hat{a}_x + 0\hat{a}_y + 4\hat{a}_z)$

$\therefore \vec{\rho} = 3\hat{a}_x - 4\hat{a}_z$

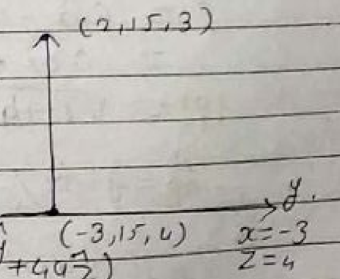
$\therefore |\rho| = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = 5.$

$\therefore \hat{a}_\rho = \frac{\vec{\rho}}{|\rho|} = \frac{3\hat{a}_x - 4\hat{a}_z}{5}$

$$\therefore E = \frac{\cancel{25\times10^{-9}}\cdot(3\hat{a}_x - 4\hat{a}_z)\times18\times10^{9}}{\cancel{18\times10^{9}}\cdot\cancel{25}}$$

$\therefore \boxed{E = 54\hat{a}_x - 72\hat{a}_z}$ V/m -- (1) Ans.

ii) E at (2, 15, 3)?




$\therefore E = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho$

$\vec{\rho} = 2\hat{a}_x + 15\hat{a}_y + 3\hat{a}_z - (-3\hat{a}_x + 15\hat{a}_y + 4\hat{a}_z)$

$\therefore \vec{\rho} = 5\hat{a}_x + 0\hat{a}_y - \hat{a}_z$

$\therefore |\rho| = \sqrt{5^2+(-1)^2} = \sqrt{26}$

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$\therefore \hat{a}_P = \dfrac{5\hat{a}_x - \hat{a}_z}{\sqrt{26}}$

$\therefore E = \dfrac{25\times10^{-9} \times 18\times10^{9} \cdot (5\hat{a}_x - \hat{a}_z)}{\sqrt{26}\cdot\sqrt{26}}$

$\therefore E = 17.307\ (5\hat{a}_x - \hat{a}_z)$

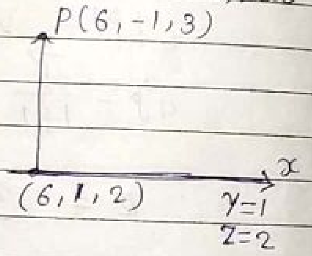
$\therefore \boxed{E = 86.538\ \hat{a}_x - 17.307\ \hat{a}_z}$ V/m

Q. 2) A line charge density of 24 nC/m is located in free space on the line $y=1$ & $z=2$
i) Find E at $P(6,-1,3)$ ii) A point charge Q of 5 μC is located at $A(-3,4,1)$. Find E at point P. iii) Find total Electric field intensity at point P.

Soln:
i) E at $P(6,-1,3)$.

$\rho_L = 24$ nC/m.

$\dfrac{1}{2\pi\epsilon_0} = 18\times10^{9}$



$\therefore E = \dfrac{\rho_L}{2\pi\epsilon_0 \rho}\ \hat{a}_P$

$\vec{\rho} = 6\hat{a}_x - \hat{a}_y + 3\hat{a}_z - (6\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$

$= 0\hat{a}_x + 2\hat{a}_y + \hat{a}_z$

$|\rho| = \sqrt{1^2+4+0} = \sqrt{5}$

$\therefore \hat{a}_P = 2\hat{a}_y + \hat{a}_z / \sqrt{5}$

$\therefore E = \dfrac{24\times10^{-9}\times18\times10^{9}\ (\hat{a}_z - 2\hat{a}_y)}{\sqrt{5}\times\sqrt{5}} = \cancel{432\ \hat{a}_z}$

$= 86.4\ (-2\hat{a}_y + \hat{a}_z) = \underline{-172.8\ \hat{a}_y + 86.4\ \hat{a}_z}$ V/m

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ii) $E = \dfrac{Q_S}{4\pi\epsilon_0 r^2}\ \hat{a}_r$ $\qquad Q = 5\times10^{-6}$ C.

$Q_S = A(-3,4,1)$ source
$P = 6,-1,3$. Des

$\therefore \vec{r} = 6\hat{a}_x - \hat{a}_y + 3\hat{a}_z - (-3\hat{a}_x + 4\hat{a}_y + \hat{a}_z)$

$= 9\hat{a}_x - 5\hat{a}_y + 2\hat{a}_z$

$\therefore |r| = \sqrt{9^2 + (-5)^2 + (2)^2} = \sqrt{81+25+4} = \sqrt{110}$

$\therefore \hat{a}_r = \dfrac{9\hat{a}_x - 5\hat{a}_y + 2\hat{a}_z}{\sqrt{110}}$

$\therefore E = \dfrac{5\times10^{-6} \times 9\times10^{9}\ (9\hat{a}_x - 5\hat{a}_y + 2\hat{a}_z)}{110\sqrt{110}}$

$\therefore E = 39.005\ (9\hat{a}_x - 5\hat{a}_y + 2\hat{a}_z)$

$\therefore \boxed{E_Q = 351.047\ \hat{a}_x - 195.025\ \hat{a}_y + 78.01\ \hat{a}_z}$ V/m.

iii) $E_T = E_Q + E_P$

$= 351.047\ \hat{a}_x - 195.025\ \hat{a}_y + 78.01\ \hat{a}_z + (-172.8\ \hat{a}_y + 86.4\ \hat{a}_z)$

$\boxed{E_T = 351.047\ \hat{a}_x - 367.825\ \hat{a}_y + 164.41\ \hat{a}_z}$ V/m

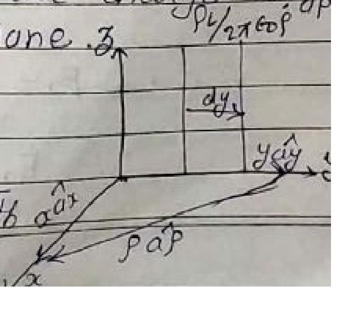
13-01-17 MGMP

* Electric Field Intensity Due to Surface Charge

$E = \dfrac{\rho_L}{2\pi\epsilon_0 \rho}\ \hat{a}_P$ Consider an ∞ uniform surface charge of density $\rho_S$ C/m² on y-z plane.

$\therefore dE = \dfrac{\rho_L}{2\pi\epsilon_0 \rho}\cdot \hat{a}_P$

$\therefore \rho_S = \dfrac{\text{charge}}{\text{surface}} = \dfrac{\text{charge}}{\text{length}\times\text{breadth}}$



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$\therefore \rho_s = \dfrac{\text{charge}}{\text{length}} \times \dfrac{1}{\text{breadth}}$

$\therefore \rho_s = \rho_L \times 1/dy \Rightarrow \boxed{\rho_L = \rho_s dy}$

$\therefore dE = \dfrac{\rho_s dy}{2\pi\epsilon_0 \rho} \hat{a_\rho}$

$\therefore$ By vector law of addition.

$y\hat{a_y} + \rho\hat{a_\rho} = x\hat{a_x}$

$\rho\hat{a_\rho} = x\hat{a_x} - y\hat{a_y}$

$\therefore |\rho| = \sqrt{x^2+y^2}$

$\therefore \hat{a_\rho} = \dfrac{\rho}{|\rho|} = \dfrac{x\hat{a_x} - y\hat{a_y}}{\sqrt{x^2+y^2}}$

$\therefore dE = \dfrac{\rho_s dy}{2\pi\epsilon_0 \sqrt{x^2+y^2}} \cdot \dfrac{(x\hat{a_x} - y\hat{a_y})}{\sqrt{x^2+y^2}}$

i.e. $dE = \dfrac{\rho_s dy\,(x\hat{a_x} - y\hat{a_y})}{2\pi\epsilon_0 (\sqrt{x^2+y^2})^2} = \dfrac{\rho_s dy\,(x\hat{a_x} - y\hat{a_y})}{2\pi\epsilon_0 (x^2+y^2)}$

Because the surface charge is $\infty$ in y-direction so we can neglect it.

$\therefore dE = \dfrac{\rho_s dy\,(x\hat{a_x})}{2\pi\epsilon_0 (x^2+y^2)}$

$\therefore dE_x = \dfrac{\rho_s dy\, x}{2\pi\epsilon_0 (x^2+y^2)}$

$\therefore E_x = \int_{-\infty}^{\infty} dE_x$

$\therefore E_x = \int_{-\infty}^{\infty} \dfrac{\rho_s dy x}{2\pi\epsilon_0 (x^2+y^2)} = \dfrac{\rho_s x}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \dfrac{dy}{x^2+y^2}$

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$\therefore E_x = \dfrac{\rho_s x}{2\pi\epsilon_0} \dfrac{1}{x} \left[\tan^{-1}\left(\dfrac{y}{x}\right)\right]_{-\infty}^{\infty} \quad \left(\because \int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)\right)$

$\therefore E_x = \dfrac{\rho_s}{2\pi\epsilon_0} \cdot \tan^{-1}(\infty) - \tan^{-1}(-\infty)$

$\therefore E_x = \dfrac{\rho_s}{2\pi\epsilon_0} (\pi/2 + \pi/2) = \dfrac{\rho_s}{2\cancel{\pi}\epsilon_0} \cancel{\pi}$

$\therefore E = \dfrac{\rho_s}{2\epsilon_0} \hat{a_x}$

$\hat{a_x}$ is the unit vector Normal to the surface y-z. Therefore the direction of E in general is given by a unit normal vector ($\hat{a_N}$).

$\therefore \boxed{E = \dfrac{\rho_s}{2\epsilon_0} \hat{a_N}}$

NN9MP ★

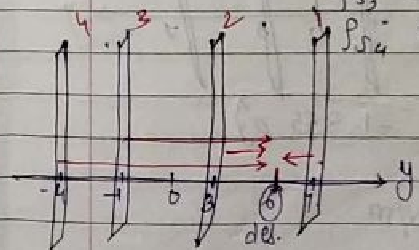
Q.1) Four $\infty$ sheets of charge are located as follows:- $20\ pC/m^2$ at $y=7$, $-8\ pC/m^2$ at $y=3$, $6\ pC/m^2$ at $y=-1$, $-18\ pC/m^2$ at $y=-4$.
Find E at i) (2, 6, +4), ii) (0,0,0), iii) ($10^6$, $10^6$, $10^6$) iv) (-1, -1.1, 5).

Soln: Given: $\rho_{s_1} = 20\,pC/m^2$ at $y=7$

$\rho_{s_2} = -8\,pC/m^2$ at $y=3$

$\rho_{s_3} = 6\,pC/m^2$ at $y=-1$

$\rho_{s_4} = -18\,pC/m^2$ at $y=-4$




$\therefore E = \dfrac{\rho_{s1}}{2\epsilon_0}\hat{a_{N_1}} + \dfrac{\rho_{s2}}{2\epsilon_0}\hat{a_{N_2}} + \dfrac{\rho_{s3}}{2\epsilon_0}\hat{a_{N_3}} + \dfrac{\rho_{s4}}{2\epsilon_0}\hat{a_{N_4}}$

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$\therefore E = \frac{1}{2\epsilon_0}\left[20\times10^{-12}\,\hat{a}_{N_1} - 8\times10^{-12}\,\hat{a}_{N_2} + 6\times10^{-12}\,\hat{a}_{N_3} - 18\times10^{-12}\right.$

$\therefore E = \frac{10^{-12}}{2\epsilon_0}\left[20\,\hat{a}_{N_1} - 8\,\hat{a}_{N_2} + 6\,\hat{a}_{N_3} + 18\,\hat{a}_{N_4}\right]$ --- (1)

$\therefore$ E ≠ To reach upto destination if we have to go forward then consider the direction +ve and if we have to go reverse then consider the direction -ve.

$\therefore a_{N_1} = -a_y,\ a_{N_2} = a_y,\ a_{N_3} = a_y,\ a_{N_4} = a_y.$

$\therefore$ eqⁿ (1) becomes

$E = \frac{10^{-12}}{2\epsilon_0}\left[-20\hat{a}_y - 8\hat{a}_y + 6\hat{a}_y - 18\hat{a}_y\right]$

$\therefore E = \frac{10^{-12}}{2\epsilon_0}(-40\hat{a}_y)$ $\quad \frac{1}{2\epsilon_0} = 18\pi\times10^{9}$

$\therefore E = 10^{-12}\times 18\pi\times10^{9}\,(-40\hat{a}_y)$

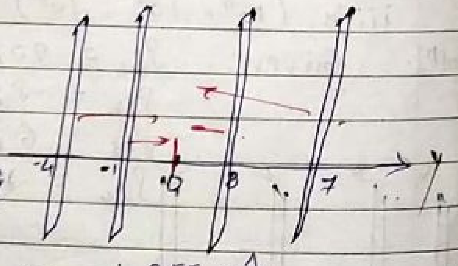
$\therefore E = 0.56.52\times10^{-12}\ (-40\hat{a}_y)$

$\therefore E = -2.260\,\hat{a}_y\ V/m$

ii) E at (0, 0, 0)

$a_{N_1} = -a_y \quad a_{N_3} = a_y$

$a_{N_2} = -a_y \quad a_{N_4} = a_y.$



$\therefore E = \frac{+10^{-12}}{2\epsilon_0}\left[-20\hat{a}_y + 8\hat{a}_y + 6\hat{a}_y - 18\hat{a}_y\right]$

$\therefore E = \frac{1}{2\times8.854}\times\left[-24\hat{a}_y\right] = -1.355\,\hat{a}_y$

$\therefore E = -1.355\,\hat{a}_y\ V/m$

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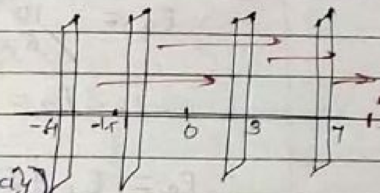
iii) E at (-1, -1.1, 5)

$a_{N_1} = -a_y,\ a_{N_2} = -a_y$

$a_{N_3} = -a_y,\ a_{N_4} = a_y.$



$\therefore E = \frac{10^{-12}}{2\epsilon_0}\left[-20\hat{a}_y + 8\hat{a}_y - 6\hat{a}_y - 18\hat{a}_y\right]$

$\therefore E = \frac{1}{2\times8.854}(-46\hat{a}_y) = -2.597\,\hat{a}_y$

iv) E at $(10^6, 10^6, 10^6)$ bcz $10^6 = 1000000$ (+ve

$a_{N_1} = a_y,\ a_{N_2} = a_y,\ a_{N_3} = a_y,\ a_{N_4} = a_y$

$\therefore E = \frac{10^{-12}}{2\epsilon_0}(+20\hat{a}_y - 8\hat{a}_y + 6\hat{a}_y - 18\hat{a}_y)$

$\therefore E = \frac{1}{2\times8.854}(0) = 0.$

Q.2. 2 ∞ sheets of uniform charge density $\rho_s = \frac{10^{-9}}{6\pi}$ c/m are located at z = -5 & y = -5 m. Find E at i) (4, 2, 2). if the line charge of 5μC is located at y = 0, z = 0.

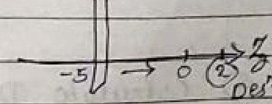
Solⁿ: Given: $\rho_{s_{1+2}} = \frac{10^{-9}}{6\pi}\ C/m^2 = 0.053\ pC/m^2$.

at z = -5 ($\rho_{s_1}$) & y = -5 m ($\rho_{s_2}$).

i)

$E_1$ at (4, 2, 2).

$\rho_{s_1} = 0.053\ pC/m^2$



$\therefore E_1 = \frac{\rho_{s_1}}{2\epsilon_0}\,\hat{a}_N$

$\therefore E_1 = \frac{10^{-9}}{6\pi}\cdot 18\pi\times10^{9}\ \hat{a}_z$

$\therefore E_1 = 3\,\hat{a}_z\ V/m.$

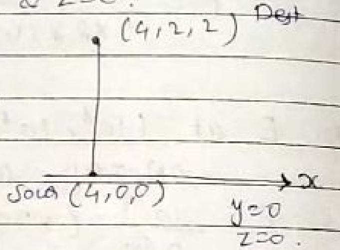
$\therefore E_2 = \frac{10^{-9}}{6\pi} \, 18\pi \times 10^{9} \, \hat{a_y}$

$\therefore E_2 = 3\hat{a_y}$ V/m.

$\therefore E_{\rho_s} = E_1 + E_2 = 3\hat{a_y} + 3\hat{a_z}$

ii) $\rho_L = 5\mu C$ at $y=0$ & $z=0$.

$\therefore E = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho}$




$\therefore \vec{\rho} = \cancel{4\hat{a_x}} + 2\hat{a_y} + 2\hat{a_z} - \cancel{4\hat{a_x}}$

$\therefore \vec{\rho} = 2\hat{a_y} + 2\hat{a_z}$

$\therefore |\rho| = \sqrt{8} = 2\sqrt{2}$

$\therefore \hat{a_\rho} = \frac{2\hat{a_y} + 2\hat{a_z}}{2\sqrt{2}}$

$\therefore E = \frac{5\times10^{-6} \times 18\times10^{9}}{2\sqrt{2} \cdot 2\sqrt{2}} (2\hat{a_y} + 2\hat{a_z})$

$\therefore E = 11250 \, (2\hat{a_y} + 2\hat{a_z})$

$\therefore E_{\rho_L} = 22500 \, \hat{a_y} + 22500 \, \hat{a_z}$ V/m.

$\therefore E = \rho_{E_L} + \rho_{E_S}$

$= 22500 \, \hat{a_y} + 22500 \, \hat{a_z} + 3\hat{a_y} + 3\hat{a_z}$

$\therefore \boxed{E = 22503 \, \hat{a_y} + 22503 \, \hat{a_z}}$ V/m.

3) Calculate D in rectangular quadinates at point P(2,-3,6) produced by (a) A point charge ($Q_A$) = 55 mC at Q(-2, 3, -6) (b) A uniform line charge ($\rho_L$) = 20 mC/m² on x-axis. (c) A uniform surface charge density ($\rho_s$) = 1.20 μC/m² on



the plane z = -5 m.

Soln: Given: P(2, -3, 6) destination

$Q_A = 55$ mC at Q(-2, 3, -6)

$E = \frac{Q_A}{4\pi\epsilon_0 r^2} \hat{a_r}$

$\therefore \vec{r} = (2\hat{a_x} - 3\hat{a_y} + 6\hat{a_z}) - (-2\hat{a_x} + 3\hat{a_y} - 6\hat{a_z})$

$\vec{r} = 4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z}$

$|r| = \sqrt{16+36+144} = 14$

$\hat{a_r} = \frac{4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z}}{14}$

$\therefore D = \epsilon_0 E = \cancel{\epsilon_0} \frac{Q_A}{4\pi\cancel{\epsilon_0} r^2} \hat{a_r}$

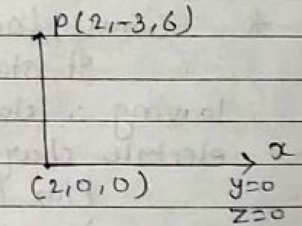
$\therefore D = \frac{Q_A}{4\pi r^2} \hat{a_r} = \frac{55\times10^{-3} \times 0.08 \, (4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z})}{(14)^3}$

$\therefore D = 1.603\times10^{-6} (4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z})$

$\therefore D_a = 6.413\hat{a_x} - 9.618\hat{a_y} + 1.9236\hat{a_z}$ μC

(b) Given: $\rho_L = 20$ mC/m²

P(2, -3, 6)




$\therefore D = \epsilon_0 E = \cancel{\epsilon_0} \frac{\rho_L}{2\pi\cancel{\epsilon_0}\rho} \hat{a_\rho}$

$\therefore D = \frac{\rho_L}{2\pi\rho} \hat{a_\rho}$

$\therefore \vec{\rho} = (2\hat{a_x} - 3\hat{a_y} + 6\hat{a_z}) - (2\hat{a_x})$

$\therefore \vec{\rho} = 0\hat{a_x} - 3\hat{a_y} + 6\hat{a_z}$

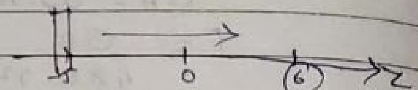
$\therefore |\rho| = \sqrt{9+36} = 3\sqrt{5}$

$\therefore \hat{a_\rho} = \frac{-3\hat{a_y} + 6\hat{a_z}}{3\sqrt{5}}$

$\therefore D_L = \frac{20\times10^{-3} \times 0.159 \, (-3\hat{a_y} + 6\hat{a_z})}{(3\sqrt{5})^2} = 70.66\times10^{-6} (-3\hat{a_y} + 6\hat{a_z})$

$\therefore D_L = -211.98\, \hat{a_y} + 423.96\, \hat{a_z} \ \mu C/m^2$

(c) Given :- $\rho_s = 120 \mu C/m^2$
$P(2, -3, 6)$



$\therefore D = \epsilon_0 E = \cancel{\epsilon_0} \dfrac{\rho_s}{2\cancel{\epsilon_0}} \hat{a_N}$

$\therefore D_s = \dfrac{\rho_s}{2} \hat{a_N}$ where $\hat{a_N} = \hat{a_z}$

$\therefore D_s = \dfrac{120 \times 10^{-6}}{2} \hat{a_z} = 60 \hat{a_z} \ \mu C/m^2$

(d) $D_T = D_Q + D_s + D_L$

$\therefore D_T = (6.413 \hat{a_x} - 9.618 \hat{a_y} + 1.9236 \hat{a_z} - 211.98 \hat{a_y} + 423.96 \hat{a_z} + 60 \hat{a_z}) \times 10^{-6}$

$D_T = 6.413 \hat{a_x} - 221.598 \hat{a_y} + 485.8836 \hat{a_z}) \times 10^{-6} \ C/m^2$

## * Gauss Law *

It states that, the total electric flux, leaving a closed surface is equal to the total electric charge enclosed by the closed surface.

Flux leaving = charged enclosed

i.e. $\psi$ leaving = $Q$ enclosed.

If $d\psi = D.ds$. , then $\psi = \int d\psi$

$\therefore \psi = \int D.ds$ --- (i)

If $Q_{enclose} = \int \rho_v . dv$

$\therefore$ Gauss Law :- $\psi$ leaving = $Q$ enclosed.

$\therefore \int D.ds = \int \rho_v . dv.$

$\psi = Q = \int D.ds = \int \rho_v . dv$



Ex. 1. Given the electric flux density $D = 0.3 r^2 \hat{a_r}$ nC/m² in free space. Find (a) E at $r = 2m$, $\theta = 25°$, $\phi = 90°$ (b) Find $Q_T$ with in the sphere $r = 3m$. (c) Find total electric flux leaving the sphere $r = 4m$.

Solⁿ :- (a) $D = \epsilon_0 E$ $\quad \therefore E = \dfrac{D}{\epsilon_0} = \dfrac{0.3 r^2 \hat{a_r} \times 10^{-9}}{8.854 \times 10^{-12}}$

$E = \dfrac{0.3 (4) \hat{a_r}}{8.854 \times 10^{-3}}$ $\quad \boxed{E = 135.53 \hat{a_r} \ V/m.}$

(b) $\therefore Q = \int D.ds.$ $\quad E = \dfrac{D}{\ }$

| | $r$ | $\theta$ | $\phi$ |
|---|---|---|---|
| dL | $dr$ | $r d\theta$ | $r \sin\theta d\phi$ |

ds $\quad r^2 \sin\theta d\theta d\phi \ \hat{a_r}$

$\therefore Q = 0.3 r^4 \times 10^{-9} \int \sin\theta d\theta d\phi$

$\therefore Q = 0.3 \times (3)^4 \times 10^{-9} \int_0^{\pi} \sin\theta d\theta . \int_0^{2\pi} d\phi$

$\therefore Q = 24.3 \times 10^{-9} (-\cos\theta)_0^{\pi} . (\phi)_0^{2\pi}$ $\quad \int \ \oint$

$\therefore Q = 24.3 \times 10^{-9} (-\cos\pi + \cos 0) . (2\pi - 0)$ $\quad r \ \theta \ \phi$

$\therefore Q = 24.3 \times 10^{-9} (4\pi)$

$\therefore \boxed{Q = 0.305 \ \mu C.}$

(c) $\therefore Q = 0.3 \times (4)^4 \times 10^{-9} (4\pi)$ $\quad E = \dfrac{Q}{4\pi\epsilon_0 r^2} \hat{a_r}$

$\therefore Q = 964.608$ nC.

$Q =$

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## * Divergance & Divergance Theorem *

$\psi_{leaving} = Q_{enclosed}$ --- (By Gauss Law)

$\int D \cdot ds = \int \rho_v \, dv$

$\Delta \cdot D = \rho_v = \text{div } \vec{D}$

$\Delta = \frac{\partial}{\partial x} \hat{a_x} + \frac{\partial}{\partial y} \hat{a_y} + \frac{\partial}{\partial z} \hat{a_z}$

$D = D_x \hat{a_x} + D_y \hat{a_y} + D_z \hat{a_z}$

$\boxed{\int D \cdot ds = \int \nabla \cdot D \, dv}$ Divergence Theorem

## * Formula for Divergence *

1) In Cartision co-ordinate System.

$\text{Div } \vec{D} \Rightarrow \nabla \cdot D = \rho_v = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

2) In Cylindrikal co-ordinate system:

$\text{Div. } \vec{D} = \nabla \cdot D = \rho_v = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} D_\phi + \frac{\partial}{\partial z} D_z$

3) In Spherical co-ordinate System :-

$\text{Div } \vec{D} = \nabla \cdot D = \rho_v = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta D_\theta) + \frac{1}{r \sin\theta} \times \frac{\partial}{\partial \phi} D_\phi$.

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Ex 1. $D = \underbrace{(2xyz - y^2)}_{D_x} \hat{a_x} + \underbrace{(x^2 z - 2xy)}_{D_y} \hat{a_y} + \underbrace{x^2 y}_{D_z} \hat{a_z}$ at $P(1, 2, 3)$.

Soln:- Given : $D_x = 2xyz - y^2$

$D_y = x^2 z - 2xy$

$D_z = x^2 y$.

$\therefore \nabla \cdot D = \rho_v = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$= \frac{\partial}{\partial x} (2xyz - y^2) + \frac{\partial}{\partial y} (x^2 z - 2xy) + \frac{\partial}{\partial z} (x^2 y)$.

$\nabla \cdot D = \rho_v = (2yz - 0) + (0 - 2x) + 0$.

$\therefore \nabla \cdot D = \rho_v = 2yz - 2x$.

$\therefore$ Put values of $x, y, z$ in the above eqn, we get,

$\therefore \nabla \cdot D = \rho_v = \underline{10}$.

2. $D = \underbrace{2\rho z^2 \sin^2\phi}_{D_\rho} \hat{a_\rho} + \underbrace{\rho z^2 \sin^2\phi}_{D_\phi} \hat{a_\phi} + \underbrace{2\rho^2 z \sin^2\phi}_{D_z} \hat{a_z}$ at $P(2, 110°, -1)$

Soln:- Given : $D_\rho = 2\rho z^2 \sin^2\phi$

$D_\phi = \rho z^2 \sin^2\phi$

$D_z = 2\rho^2 z \sin^2\phi$

$\therefore \nabla \cdot D = \rho_v = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} D_\phi + \frac{\partial}{\partial z} D_z$

$\therefore \nabla \cdot D = \rho_v = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \, 2\rho z^2 \sin^2\phi) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho z^2 \sin^2\phi) + \frac{\partial}{\partial z} (2\rho^2 z \sin^2\phi)$.

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$$\therefore \nabla\cdot D = \rho_v = \frac{1}{\rho}\frac{\partial}{\partial \rho}(2\rho^2 z^2 \sin^2\phi) + \frac{1}{\rho}\rho z^2 \frac{\partial}{\partial \phi}(\sin^2\phi) + 2\rho^2\sin^2\phi \frac{\partial}{\partial z}(z)$$

$$= \frac{1}{\cancel{\rho}} 2z^2\sin^2\phi\,(2\cancel{\rho}) + z^2\, 2\sin\phi\cos\phi + 2\rho^2\sin^2\phi\,(1)$$

$$= 4z^2\sin^2\phi + z^2\, 2\sin\phi\cos\phi + 2\rho^2\sin^2\phi$$

Put $\rho = 2 \quad \phi = 110^\circ \quad \& \quad z = -1$

$$\therefore \nabla\cdot D = \rho_v = 4(-1)^2\sin(110^\circ)^2 + (-1)^2\cdot 2\sin(110^\circ)\cos(110^\circ) + 2(2)^2\sin(110^\circ)^2$$

$$\nabla\cdot D = \cancel{10.406}\ \ 9.953$$

3. $D = (2r\sin\theta\cos\phi)\,\hat{a_r} + r\cos\theta\cos\phi\,\hat{a_\theta} - r\sin\theta\,\hat{a_\phi}$ at $P(1.5, 30^\circ, 50^\circ)$

Soln:- Given:- $D_r = 2r\sin\theta\cos\phi$

$D_\theta = r\cos\theta\cos\phi$

$D_\phi = r\sin\theta$.

$$\therefore \nabla\cdot D = \rho_v = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}D_\phi$$

$$\therefore \nabla\cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\, 2r\sin\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, r\cos\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(r\sin\theta)$$

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$$\therefore \nabla\cdot D = \frac{1}{r^2} 2\sin\theta\cos\phi \frac{\partial}{\partial r}(r^3) + \frac{1}{\cancel{r}\sin\theta}\cancel{r}\cos\phi \frac{\partial}{\partial\theta}\sin\theta.\cos\theta. + \frac{1}{r\sin\theta} r\sin\theta \frac{\partial}{\partial\phi}(1)$$

$$\therefore \nabla\cdot D = \frac{1}{\cancel{r^2}} 2\sin\theta\cos\phi\, 3\cancel{r^2} + \frac{\cos\phi}{\sin\theta} \ast \frac{1}{2}\frac{\partial}{\partial\theta} 2\sin\theta.\cos\theta + 0.$$

$$\therefore \nabla\cdot D = 6\sin\theta\cos\phi + \frac{\cos\phi}{\sin\theta}\frac{1}{2}\frac{\partial}{\partial\theta}\sin 2\theta.$$

$$\therefore \nabla\cdot D = 6\sin(30)\cos(50) + \frac{\cos(50)}{2\sin(30)}\cos 2(30).2$$

$$\therefore \boxed{\nabla\cdot D = \cancel{1.288}\ \ 2.571}$$

$--(\because \sin 2\theta = 2\cos 2\theta)$

4) $D = 2xy\,\hat{a_x} + x^2\hat{a_y}$ $C/m^2$ evaluate both sides of divergence theorem $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$

Soln:- 
$$\int D.ds = \underbrace{\int_{\substack{x=0\\(-\hat{a_x})}} + \int_{\substack{x=1\\(\hat{a_x})}}}_{dydz} + \underbrace{\int_{\substack{y=0\\(-\hat{a_y})}} + \int_{\substack{y=2\\(\hat{a_y})}}}_{dxdz} + \underbrace{\int_{\substack{z=0\\(-\hat{a_z})}} + \int_{\substack{z=3\\(\hat{a_z})}}}_{dxdy}$$

ds.

① $\therefore \int_{x=0} D.ds = \int_{x=0} (2xy\,\hat{a_x} + x^2\hat{a_y})\cdot \underbrace{dydz}_{ds}(-\hat{a_x})$

$$= \int_{x=0} -2xy\, dydz + 0.$$

$$= -2x\int_0^2 y\,dy - \int_0^3 dz.$$

$$= -2x\left(\frac{y^2}{2}\right)_0^2 \cdot (z)_0^3$$

$$= -\cancel{2}x\ (4).3 = 0 \quad --- (\because x = 0)$$

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② $\therefore \int_{x=1} D.ds = \int_{x=1} (2xy\,\hat{a_x} + x^2\hat{a_y}) \cdot dy\,dz\,(\hat{a_x})$

$= \int_{x=1} 2xy\,dy\,dz + 0.$

$= 2x \int_0^2 y\,dy \cdot \int_0^3 dz.$

$= 2x \left(\frac{y^2}{2}\right)_0^2 \cdot (z)_0^3$

$= \frac{\cancel{2}x}{\cancel{2}} (4)(3)$

$\int_{x=1} D.ds = 12 \quad --- \quad (\because x = 1).$

③ $\int_{y=0} D.ds = \int_{y=0} (2xy\,\hat{a_x} + x^2\hat{a_y}) \cdot dx\,dz\,(-\hat{a_y})$

$= \int_{y=0} 0 - x^2\,dx\,dz.$

$= -\int_0^1 x^2 dx \cdot \int_0^3 dz$

$= -\left(\frac{x^3}{3}\right)_0^1 \cdot (z)_0^3$

$= -\frac{1}{\cancel{3}} \cdot \cancel{3} = -1.$

④ $\int_{y=2} D.ds = \int_{y=2} (2xy\,\hat{a_x} + x^2\hat{a_y}) \cdot dx\,dz\,(\hat{a_y})$

$= \int_{y=2} 0 + x^2\,dx\,dz$

$= \int_0^1 x^2 dx \cdot \int_0^3 dz.$

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$\int_{y=2} D.ds = \left(\frac{x^3}{3}\right)_0^1 \cdot (z)_0^3$

$= \frac{1}{\cancel{3}} \cdot \cancel{3} = 1.$

⑤ $\int_{z=0} D.ds = \int_{z=0} (2xy\,\hat{a_x} + x^2\hat{a_y})\,dx\,dy\,(-a_z)$

$= 0.$

⑥ $\int_{z=3} D.ds = \int_{z=3} (2xy\,\hat{a_x} + x^2\hat{a_y})\,dx\,dy\,(a_z).$

$= 0.$

$\therefore \oint D.ds = 0 + 12 \cancel{-1} \cancel{+1} + 0$

$\therefore \boxed{\oint D.ds = 12}$

(i) $\nabla . D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z.$

$\therefore \nabla . D = \frac{\partial}{\partial x} 2xy + \frac{\partial}{\partial y} x^2 + 0$

$\therefore \nabla . D = 2y + 0 + 0$

$\therefore \boxed{\nabla . D = 2y.}$

$\int \nabla . D . dv = \int 2y\,dx\,dy\,dz \quad --- \quad (\because dv = dx\,dy\,dz)$

$= 2 \int_0^1 dx \cdot \int_0^2 y\,dy \cdot \int_0^3 dz.$

$= 2 (x)_0^1 \cdot \left(\frac{y^2}{2}\right)_0^2 \cdot (z)^3$

$= \frac{\cancel{2}}{\cancel{2}} \cdot 4 \cdot 3$

$\boxed{\int \nabla . D . dv = 12}$

Q. $D = (8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}$

$-a < x, y, z < a$.

Sol$^n$:- Given $D_x = 8x+4x^3$

$D_y = 2y$

$D_z = 2z$.

$$\int D.ds = \int_{x=-a} + \int_{x=a} + \int_{y=-a} + \int_{y=a} + \int_{z=-a} + \int_{z=a}$$

$(-\hat{a_x})$ $(\hat{a_x})$ $(-\hat{a_y})$ $(\hat{a_y})$ $(-\hat{a_z})$ $(\hat{a_z})$

$dydz$. $dxdz$ $dxdy$.

① $\int D.ds = \int_{x=-a} [(8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}]\, dydz\, (-\hat{a_x})$

$= \int_{x=-a} -(8x+4x^3).\,dydz.$

$= -(8x+4x^3) \int_{y=-a}^{a} dy . \int_{z=-a}^{a} dz.$

$= -(8x+4x^3)\, (y)_{-a}^{a} . (z)_{-a}^{a}.$

$= (+8a-4a^3)(a+a).(a+a)$

$= (8a-4a^3)\, 2a . 2a.$

$= \cancel{16a^3}\; 4a^2\,(8a+4a^3)$

$= 32a^3 + 16a^5$

② $\int D.ds. = \int_{x=a} [(8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}]\, dy\, dz\, (\hat{a_x})$

$= \int_{x=a} (8x+4x^3)\, dy\, dz.$

$= (8x+4x^3). \int_{-a}^{a} dy . \int_{-a}^{a} dz.$



$= (8x+4x^3)\quad (2a).(2a).$

$= (8a+4a^3)\, 4a^2$

$\int D.ds. = 32a^3 + 16^5$

⑧ $\int D.ds = \int_{y=-a} [(8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}]\, dxdz\, (-\hat{a_y})$

$= \int_{y=-a} +2y\, dx\, dz.$

$= 2y \int_{-a}^{a} dx \int_{-a}^{a} dz$

$= 2y\ (2a)(2a)$

$= (-2a)\, 4a^2$

$= -8a^3$

④ $\int D.ds = \int_{y=a} [(8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}]\, dxdz\, (\hat{a_y})$

$= \int_{y=a} -2y\ dxdz.$

$= -2y \int_{-a}^{a} dx \int_{-a}^{a} dz.$

$= -2a\ (4a^2)$

$= -8a^3$

⑤ $\int D.ds = \int_{z=-a} [(8x+4x^3)\hat{a_x} - 2y\hat{a_y} + 2z\hat{a_z}]\, dxdy\, (-\hat{a_z})$

$= \int_{z=-a} -2z\, dxdy$

$= -2z \int_{-a}^{a} dx \int_{-a}^{a} dy \quad = +2a\,(4a^2)$

$= 8a^3$

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⑥ $\int D.ds = \int_{z=a} \left[(8x+4x^3)\hat{a_x} - 2y\,\hat{a_y} + 2z\,\hat{a_z}\right] dx\,dy\,(\hat{a_z})$

$= \int_{z=a} 2z\, dx\,dy$

$= 2a\,(4a^2)$

$= 8a^3$

$\therefore \int D.ds = (32a^3 + 16a^5) + (32a^3 + 16a^5) + (\cancel{-8a^3}) + (\cancel{-8a^3})$
$\quad + \cancel{8a^3} \quad + \cancel{8a^3}$

$\boxed{\int D.ds = 64a^3 + 32a^5}$

(i) $\nabla.D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$\nabla.D. = \frac{\partial}{\partial x}(8x+4x^3) + \frac{\partial}{\partial y}(-2y) + \frac{\partial}{\partial z} 2z.$

$= \frac{\partial}{\partial x} 8x + \frac{\partial}{\partial x} 4x^3 - \cancel{2}\left(\cancel{y^2/2}\right) + \cancel{2}\left(\cancel{z^2/2}\right)$

$= \cancel{\frac{48x^2}{2}} + \cancel{\frac{4x^4}{4}} - y^2 + z^2 \quad \boxed{8 + 12x^2}$

$\boxed{\cancel{\nabla.D.} = \cancel{(4x^2 + x^4) - y^2 + z^2}}$

$\therefore \int \nabla.D\, dv = \int_a (\cancel{4x^2 + x^4) - y^2 + z^2 . dx\,dy\,dz}$

$= \cancel{4\int_{-a}^{a} x^2 dx + \int_{-a}^{a} x^4 dx - \int_{-a}^{a} y^2 dy + \int_{-a}^{a} z^2 dz}$

$= \cancel{4(2x)}_{-a}^{a}$

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$\therefore \int \nabla.D\, dv = \int_a (8 + 12x^2)\, dx\, dy\, dz$

$= 8\int_{-a}^{a} dx + 12\int_{-a}^{a} x^2 dx . \int_{-a}^{a} dy . \int_{-a}^{a} dz.$

$= 8(x)_{-a}^{a} + 12\left(\frac{x^3}{3}\right)_{-a}^{a} . (2a).(2a).$

$= \left[8(2a) + \frac{\cancel{12}^{4}}{\cancel{3}_{1}}\left(a^3 - (-a)^3\right)\right]. 4a^2$

$= (16a + 8a^3)\, 4a^2.$

$\boxed{\int \nabla.D.\, dv. = 64a^3 + 32a^5}$

Hence proved.

Ex. 3. $D = \frac{10\rho^3}{4}\hat{a_\rho}$ , $\rho = 1$, $\rho = 2$, $z = 0$ & $z = 10$.

Soln: Given: $D_\rho = \frac{10}{4}\rho^3$

$D_\phi = 0$

$D_z = 0.$ $\quad 0 - 2\pi$

$\int D.ds = \int_{\rho=1} + \int_{\rho=2} + \int_{\phi=0} + \int_{\phi=2\pi} + \int_{z=0} + \int_{z=10}$

$(-\hat{a_\rho}) \quad (\hat{a_\rho}) \quad (-\hat{a_\phi}) \quad (\hat{a_\phi}) \quad (-\hat{a_z}) \quad (\hat{a_z})$

$\underline{\underline{ds}} \quad \underline{\rho\, d\phi\, dz.} \quad d\rho\, dz. \quad \rho\, d\rho\, d\phi$

① $\therefore \int D.ds = \int_{\rho=1} \frac{10}{4}\rho^3 \hat{a_\rho} . \underline{\rho\, d\phi\, dz}\, (-\hat{a_\rho})$

$= \int_{\rho=1} -\frac{10}{4}\rho^3 . \rho\, d\phi\, dz.$

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$$\therefore \int D.ds = \frac{10}{4}\rho^4 \int_{\phi=0}^{2\pi} d\phi \cdot \int_0^{10} dz$$

$$= -\frac{10}{4}\rho^4 (\phi)_0^{2\pi} (z)_0^{10}$$

$$= -\frac{10}{4}\rho^4 (2\pi - 0)(10-0)$$

$$= -\frac{10}{4} \times 20\pi \quad --- \quad (\because \rho = 1)$$

$$= -157$$

② $$\int D.ds = \int_{\rho=2} \frac{10}{4}\rho^3 \hat{a_\rho}\ \rho\, d\phi\, dz\, (\hat{a_\rho})$$

$$= \frac{10}{4}\rho^4 \int_0^{2\pi} d\phi \cdot \int_0^{10} dz$$

$$= \frac{10}{4}\rho^4 (\phi)_0^{2\pi} \cdot (z)_0^{10}$$

$$= \frac{10}{4} \times \frac{16}{4} (2\pi)(10) \quad --- \quad (\because \rho = 2)$$

$$= 40 \times 20\pi = 2512$$

$$\therefore \int D.ds = 2512$$

$$\therefore \int D.ds = -157 + 2512 = 2355$$

$$\therefore \nabla \cdot D = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho D_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} D_\phi + \frac{\partial}{\partial z} D_z$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\left(\frac{10}{4}\rho^3\right) + 0$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{10}{4}\rho^4\right)$$

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$$= \frac{1}{\rho}\, 4\, \frac{10}{4}\rho^3$$

$$\nabla \cdot D = 10(\rho^2)$$

$$\int \nabla \cdot D\, dv = \int 10(\rho^2)\, d\rho\, \rho\, d\phi\, dz$$

$$= 10\int_1^2 \rho^3 d\rho \cdot \int_0^{2\pi} d\phi \cdot \int_0^{10} dz$$

$$= 10\left(\frac{\rho^4}{4}\right)_1^2 \cdot (\phi)_0^{2\pi} \cdot (z)_0^{10}$$

$$= \frac{10}{4}(16-1)\cdot(2\pi)(10)$$

3.14

$$= 2355$$

4. $D = \frac{5r^2}{4}\hat{a_r}$ $\quad r = 1, \ r = 2.$

Soln:

$$D_r = \frac{5r^2}{4}$$

$$D_\theta = 0$$

$$D_\phi = 0$$

$$\int D.ds = \int_{\substack{r=1 \\ (-\hat{a_r})}} + \int_{\substack{r=2 \\ (\hat{a_r})}} + \int_{\substack{\theta=0 \\ (-\hat{a_\theta})}} + \int_{\substack{\theta=\pi \\ (a_\theta)}} + \int_{\substack{\phi=0 \\ (-a_\phi)}} + \int_{\substack{\phi=2\pi \\ (+a_\phi)}}$$

| | | | | | |
|---|---|---|---|---|---|
| | $r$ | $\theta$ | $\phi$ | | |
| dl | $dr$ | $r d\theta$ | $r\sin\phi$ | | |
| ds | $r^2\sin\theta\, d\theta\, d\phi$ | $dr\, r\sin\theta\, d\phi$ | $r\, dr\, d\theta$ | | |

$$\int D.ds = \int_{r=1} \frac{5r^2}{4}\hat{a_r}\ r^2 \sin\theta\, d\theta\, d\phi (-\hat{a_r})$$

$$\int D.ds = \int_{r=1} -\frac{5r^2}{4} r^2 \sin\theta\, d\theta\, d\phi$$

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$$= -\frac{5r^2}{4} r^2 \int_0^{\pi} \sin\theta \, d\theta \cdot \int_0^{2\pi} d\phi$$

$$= -\frac{5r^4}{4} (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi}$$

$$= -\frac{5r^4}{4} (-\cos\pi + \cos(0)) \cdot (2\pi)$$

$$= -\frac{5}{4} (1+1) \cdot 2\pi$$

$$= -\frac{5}{\cancel{4}} \cancel{4}\pi = -5\pi = -15.7$$

(2) $\oint D.ds = \int_{r=2} \frac{5r^2}{4} \hat{ar} \cdot r^2 \sin\theta d\theta d\phi \, (\hat{ar})$

$$= \int_{r=2} \frac{5r^2}{4} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{5r^4}{4} \int_0^{\pi} \sin\theta \, d\theta . \int_0^{2\pi} d\phi$$

$$= \frac{5 \times \cancel{16}4}{\cancel{4}} (-\cos\theta)_0^{\pi} \cdot (\phi)_0^{2\pi}$$

$$= 20 \left(-\cos\pi + \cos(0)\right) \cdot (2\pi)$$

$$= 20 \cdot 4\pi = 251.2.$$

$$\therefore \oint D.ds = -15.7 + 251.2$$

$$\boxed{\oint D.ds = 235.5}$$

$$\therefore \nabla . D = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} \sin\theta D_\theta + \frac{1}{r\sin\theta} \frac{\partial}{\partial\phi} D\phi$$

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$$\therefore \nabla . D = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{5r^2}{4} \cancel{dr}\right) + 0.$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^4 \frac{5}{4}\right)$$

$$= \frac{1}{\cancel{r^2}} \cancel{4} \frac{5}{\cancel{4}} r^{\cancel{3}}$$

$$= 5r.$$

$$\therefore \int \nabla . D . dv = \int 5r \; r^2 dr \sin\theta d\theta d\phi$$

$$= 5 \int_1^2 r^3 dr . \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$$

$$= 5 \left(\frac{r^4}{4}\right)_1^2 . (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi}$$

$$= \frac{5}{4}(16-1) . (2)(2\pi)$$

$$= \frac{5}{\cancel{4}} \times 15 \times \cancel{4}\pi$$

$$= 75 \times 3.14$$

$$\boxed{\int \nabla . D . dV. = 235.5}$$

Hence proved.

5. $D = \frac{2\cos\theta}{r^3} \hat{ar} + \frac{\sin\theta}{r^3} \hat{a\theta}$ $1 < r < 2$, $0 < \theta < \pi/2$, $0 < \phi < \pi/2$.

Soln:- Given: $D_r = \frac{2\cos\theta}{r^3}$

$$D_\theta = \frac{\sin\theta}{r^3}$$

$$D\phi = 0$$

$$\oint D.ds = \int_{r=1} + \int_{r=2} + \int_{\theta=0} + \int_{\theta=\pi/2} + \int_{\phi=0} + \int_{\phi=\pi/2}$$

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$$\int D.ds = \int_{r=1} \frac{2\cos\theta}{r^3}\hat{a_r} + \frac{\sin\theta}{r^3}\hat{a_\theta}\; r^2\sin\theta\, d\theta\, d\phi\; (-\hat{a_r})$$

$$= \int_{r=1} -\frac{2\cos\theta}{r^{\cancel{3}}}\, \cancel{r^2}\sin\theta\, d\theta\, d\phi$$

$$= \frac{-\cancel{2}}{r}\int_0^{\pi/2} 2\sin\theta.\cos\theta\, d\theta \;.\int_0^{\pi/2} d\phi$$

$$= -\frac{1}{r}\int_0^{\pi/2} \sin 2\theta\, d\theta\, (\phi)_0^{\pi/2}$$

$$= +\frac{1}{r}\left(-\frac{\cos 2\theta}{2}\right)_0^{\pi/2}\left(\frac{\cancel{2}\pi}{2}\right)$$

$$= \frac{1}{\cancel{2}}\left(\cos \cancel{2}\left(\frac{\cancel{2}\pi}{\cancel{2}}\right) - \cos(0)\right).\frac{\cancel{2}\pi}{2}$$

$$= \cancel{\tfrac{1}{2}}\;\cancel{(-0.0239)\pi}$$

$$= -0.075V.$$

$$= \frac{1}{2}(-1-1)\,\pi/_2.$$

① $\int D.ds = -\pi/_2 = -1.57$

② $$\int D.ds = \int_{r=2} \frac{2\cos\theta}{r^3}\hat{a_r} + \frac{\sin\theta}{r^3}\hat{a_\theta}\; r^2\sin\theta\, d\theta\, d\phi\; (\hat{a_r})$$

$$= \int_{r=2} \frac{2\cos\theta}{r^3}\, \cancel{r^2}\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{r}\int_0^{\pi/2} 2\cos\theta.\sin\theta\, d\theta \;.\int_0^{\pi/2} d\phi$$

$$= 1/r\int_0^{\pi/2} \sin 2\theta\, d\theta.\, (\phi)_0^{\pi/2}$$

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$$= \frac{1}{r}\left(-\frac{\cos 2\theta}{2}\right)_0^{\pi/2} \cdot \pi/_2$$

$$= \frac{1}{4\cancel{2}}\left(-\cos \cancel{2}.\frac{\pi}{\cancel{2}} + 1\right).\frac{\pi}{2}$$

$$= \pi/_4 \quad 0.785.$$

③ $$\int D.ds = \int_{\theta=0} \frac{2\cos\theta}{r^3}\hat{a_r} + \frac{\sin\theta}{r^3}\hat{a_\theta}\; r\, dr\, \sin\theta\, d\phi\; (-\hat{a_\theta})$$

$$= \int_{\theta=0} -\frac{\sin\theta}{r^2}\, dr\, \sin\theta\, d\phi$$

$$= -\sin^2\theta \int_1^2 1/_{r^2}\, dr.\int_0^{\pi/2} d\phi$$

$$= -\sin^2\theta\left(\frac{r^2}{r^3}\right)_1^2 \cdot (\phi)_0^{\pi/2}$$

$$= 0 \quad --- \quad (\because \sin(0)=0)$$

④ $$\int D.ds = \int_{\theta=\pi/2} \frac{2\cos\theta}{r^3}\hat{a_r} + \frac{\sin\theta}{r^3}\hat{a_\theta}\; r\, dr\, \sin\theta\, d\phi\; (\hat{a_\theta})$$

$$= \int_{\theta=\pi/2} \frac{\sin\theta}{r^2}\, dr\, \sin\theta\, d\phi$$

$$= \sin^2\theta \int_1^2 1/_{r^2}\, dr.\int_0^{\pi/2} d\phi$$

$$= \sin^2(\pi/_2)\cdot\left(\cancel{\frac{r^2}{r^3}}\right)_1^2\cdot(\pi/_2)\;(-1/r)$$

$$= \cancel{0.027}\cdot\cancel{(-2(2)^{-3})_1^2}\;\cancel{(\pi/_2)}\;(1/_2 \times \pi/_2 = 0.785$$

$$= \cancel{-0.054}\;\cancel{((2)^{-3} - (1)^{-3})}\;\cancel{(\pi/_2)}$$

$$= \cancel{-0.054\;(0.125-1)\;\pi/_2} =$$

$\int D.dl = -1.57 + 0.785 + 0.785 \doteq 0$

$$\nabla.D = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 D_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta D_\theta + 0$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{2\cos\theta}{r^3} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta \frac{\sin\theta}{r^3}$$

$$= \frac{1}{r^2} 2\cos\theta \frac{\partial}{\partial r}\frac{1}{r} + \frac{1}{r\sin\theta} r^3 \frac{\partial}{\partial\theta}\sin^2\theta$$

$$= \frac{1}{r^2} 2\cos\theta (-r)^{-2} + \frac{r^2}{\sin\theta}\frac{\partial}{\partial\theta} \quad 1+\cos^2\theta.$$

$$= -\frac{2\cos\theta}{r^4} + \frac{r^2}{\sin\theta} 2\cos 2\theta$$

$$= -\left(\frac{2\cos\theta}{r^2} - \frac{1}{\sin\theta} 2.\cos 2\theta\right)$$

$$\int \nabla.D.dv = \int -\frac{2\cos\theta}{r^4} + \frac{r^2}{\sin\theta} 2.\cos 2\theta) r^2 dr \sin\theta d\theta d\phi$$

$$= \int \frac{-2\cos\theta}{r^4} \times r^2 dr + \int_{\pi/2}^{\pi/2} \frac{r^2}{\sin\theta} 2\cos 2\theta \sin\theta d\theta . \int_0^{\pi/2} d\phi$$

$$= -2\cos\theta \int dr + r^2 \int_0^{\pi/2} 2\cos 2\theta d\theta . \int d\phi$$

Q.1) Columbs Law (4 M).
Q.2) Gauss Law (4 M).
Q.3) Electric field intensity due to line charge. [7]
4) Electric Field Intensity due to surface charge. [7]
5) Numerical of Surface Charge.

$$= -2\cos\theta (r)^2_1 + 2r^2 (2\sin 2\theta)_0^{\pi/2} (\pi/2)$$

$$= -2\cos\theta (1) + 4r^2 (\sin \pi/2 - 0) . \pi/2$$

$$\doteq -2(2) + 4(4-1) . \pi/2$$

$$= -4 + 12 . \pi/2 =$$

UNIT- 2. Premitivity

MAGNETOSTATICS

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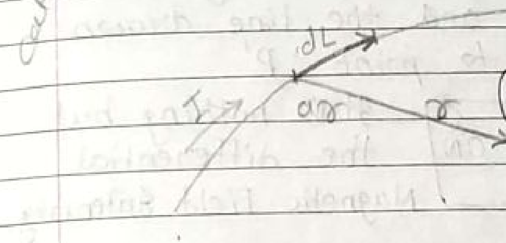
MMIMP

## * Biot Savart's Law * [7 M]



$D = \epsilon_0 E$ • Electric field.
$\epsilon = \epsilon_0 \epsilon_r$ E D
(Permitivity) Intensity Density.
• Magnetic Field.
$B = \mu_0 H$ H B
$\mu = \mu_0 \mu_r$. Intensity Density
(Permiability).

The Biot Savart's Law states that, " At any point 'P' magnitude of the magnetic field intensity produced by the differential element is proportional to the product of the current and the magnitude of the differential length". $\boxed{dH \propto IdL}$

It is inversly proportional to the square of the distance from the differential element to point 'P'. $\boxed{dH \propto 1/r^2}$

It is directly proportional to the sine of the angle lying beth the felament & a line connecting the felament to the point 'P'.

$$\boxed{dH \propto \sin\theta}$$

$$\therefore dH \propto \frac{IdL \sin\theta}{r^2}$$

$$\therefore dH = K \frac{IdL\sin\theta}{r^2} \quad \text{where } K = 1/4\pi$$

$$\therefore \boxed{dH = \frac{IdL\sin\theta}{4\pi r^2}}$$

UNIT - 2.
Premitivity
MAGNETOSTATICS



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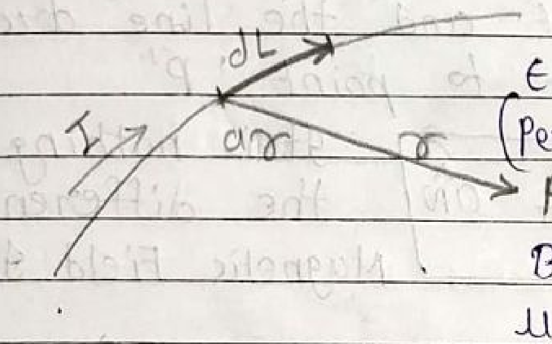
## * Biot Savart's Law * [7 M]




$D = \epsilon_0 E$ • Electric field.
$\epsilon = \epsilon_0 \epsilon_r$ E D
(Permitivity.) Intensity Density.
• Magnetic Field.
$B = \mu_0 H$ H B
$\mu = \mu_0 \mu_r$. Intensity Density.
(Permiability).

The Biot Savart's Law States that, " At any point 'P' magnitude of the magnetic field intensity produced by the differential element is proportional to the product of the current and the magnitude of the differential length".

$$\boxed{dH \propto I dL}$$

It is inversly proportional to the square of the distance from the differential element to point 'P'.

$$\boxed{dH \propto 1/r^2}$$

It is directly proportional to the sine of the angle lying betn the felament & a line connecting the felament to the point 'P'.

$$\boxed{dH \propto \sin\theta}$$

$$\therefore dH \propto \frac{I dL \sin\theta}{r^2}$$

$$\therefore dH = k \frac{I dL \sin\theta}{r^2} \quad \text{where } k = 1/4\pi$$

$$\therefore \boxed{dH = \frac{I dL \sin\theta}{4\pi r^2}}$$

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The direction of the magnetic field intensity is normal to the plane containing the differential felament and the line drawn from the felament to point 'P'.

$$\boxed{dH = \frac{I\,dL\,\sin\theta}{4\pi r^2}\,\hat{a_N}}$$ It is nothing but the differential Magnetic Field Intensity.

$$\therefore dH = \frac{I\,dL \cdot 1 \cdot \sin\theta}{4\pi r^2}\,\hat{a_N}$$

$$\therefore dH = \frac{I\,dL \cdot \hat{a_r}\,\sin\theta}{4\pi r^2}\,\hat{a_N} \quad \text{---} (\because 1 = \hat{a_r})$$

$$\therefore dH = \frac{I\,dL \times \hat{a_r}}{4\pi r^2} \quad \text{---} (\because A \times B = |A|\,|B| \sin\theta\,\hat{a_N})$$

$$\therefore H = \int dH = \int \frac{I\,dL \times \hat{a_r}}{4\pi r^2} \ \text{A/m}.$$

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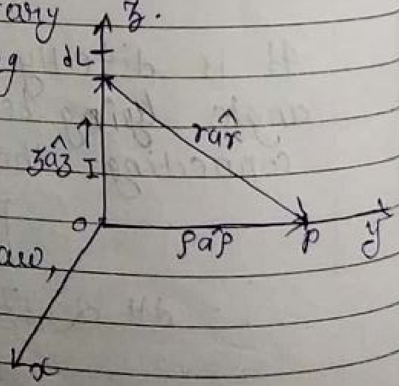
## * Magnetic Field Intensity due to an $\infty$ filament carry current 'I' in it *

Consider an $\infty$ felamentary conductor along z-axis carrying current of 'I' amperes in +ve z direction.




Consider the field point (deth) from Biot Savart Law, we have,

$$dH = \frac{I\,dL \times \hat{a_r}}{4\pi r^2}$$

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$\therefore$ By vector Law of addition,

$$z\hat{a_z} + r\hat{a_r} = \rho\hat{a_\rho}$$

$$\therefore r\hat{a_r} = \rho\hat{a_\rho} - z\hat{a_z}$$

$$\therefore \hat{a_r} = \frac{\rho\hat{a_\rho} - z\hat{a_z}}{\sqrt{\rho^2+z^2}}$$

$$|r| = \sqrt{\rho^2+z^2}$$

$$\therefore I\,dL = I\,dz\,\hat{a_z}$$

$$\therefore dH = \frac{I\,dz\,\hat{a_z} \times (\rho\hat{a_\rho} - z\hat{a_z})}{4\pi(\rho^2+z^2)\sqrt{\rho^2+z^2}}$$

$$\therefore dH = \frac{I\,dz\,\hat{a_z} \times (\rho\hat{a_\rho} - z\hat{a_z})}{4\pi(\rho^2+z^2)^{3/2}}$$

$$\therefore dH = \frac{I\,dz\,\rho\,\hat{a_\phi}}{4\pi(\rho^2+z^2)^{3/2}} \quad \text{---} \left(\because \begin{array}{l}\hat{a_x} \times \hat{a_x} = 0 \ \& \\ \hat{a_z} \times \hat{a_\rho} = \hat{a_\phi}\end{array}\right)$$

$$\therefore H = \int_{z=-\infty}^{z=\infty} dH.$$

$$\therefore H = \int_{-\infty}^{\infty} \frac{I\,dz\,\rho\,a_\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

$$\therefore z = \rho\tan\theta \qquad dz = \rho\sec^2\theta\,d\theta.$$

$$\therefore H = \int_{-\pi/2}^{\pi/2} \frac{I\,dz\,\rho\,a_\phi}{4\pi(\rho^2+\rho^2\tan^2\theta)^{3/2}}$$

$$\therefore H = \frac{I\rho\,a_\phi}{4\pi}\int_{-\pi/2}^{\pi/2} \frac{\cancel{dz}\ \rho\sec^2\theta\,d\theta}{(\rho^2\sec^2\theta)^{3/2}}$$

$$\therefore H = \frac{I\rho\,a_\phi}{4\pi\,\cancel{\rho}}\int_{-\pi/2}^{\pi/2} \cancel{dz}\ \frac{\rho\sec^2\theta\,d\theta}{\rho^3\sec^3\theta}$$

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$$dH = \frac{I\,\hat{a}_\phi}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{1}{\rho}\,\frac{1}{\sec\theta}\,d\theta.$$

$$\therefore dH_\phi = \frac{I}{4\pi}\frac{1}{\rho}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta.$$

$$\therefore dH_\phi = \frac{I}{4\pi\rho}\left(+\sin\theta\right)_{-\pi/2}^{\pi/2}$$

$$\therefore dH_\phi = \frac{I}{4\pi\rho}\left(+\sin\pi/2 - \sin(-\pi/2)\right)$$

$$\therefore dH_\phi = \frac{I}{4\pi\rho}\left(+1+1\right)$$

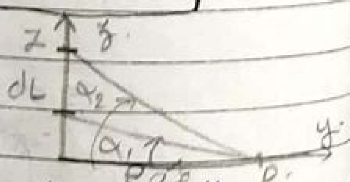
$$\boxed{H = \frac{I}{2\pi\rho}\,\hat{a}_\phi}\ A/m.$$

where, $\hat{a}_\phi = a_z \times \hat{a}_\rho$

$\therefore$ The equation for finite felament *

$$\boxed{H = \frac{I}{4\pi\rho}\left(\sin(\alpha_2) - \sin(\alpha_1)\right)\hat{a}_\phi}\ A/m.$$

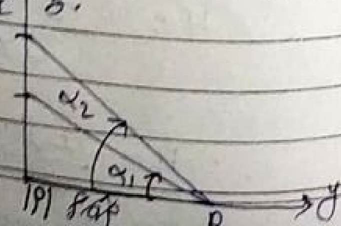
where, $\hat{a}_\phi = a_z \times \hat{a}_\rho$



* Rules :

where $\alpha_1$ is the angle made by the line segment joining lower end of felament to point 'p'

- $\alpha_1$ & $\alpha_2$ both are +ve



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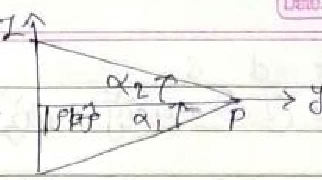
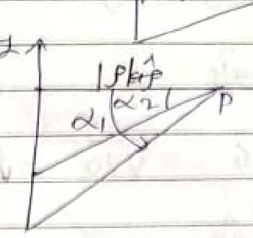
ii) $2^{nd}$ case :

- $\alpha_2$ is +ve & $\alpha_1$ is -ve



iii) $3^{rd}$ case :



$\alpha_1$ & $\alpha_2$ both are -ve.

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A NUMERICAL

Ex.1 A current felament carrying 15 Amp. in $\hat{a}_z$ direction lies along the entire z-axis. Find H in cartesion co-ordinates at (a) $P(20,0,4)$ (b) $Q(2,-4,4)$.

Soln :- $H = \frac{I}{2\pi\rho}\,\hat{a}_\phi$

$\therefore$ Given : $I = 15$ Amp. $\hat{a}_z = \hat{a}_z$

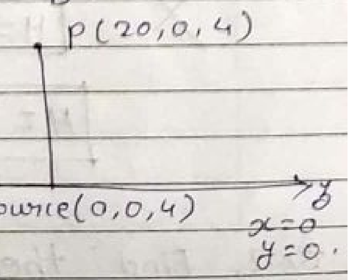
$\therefore \rho = d - S.$

$= (20\hat{a}_x + 4\hat{a}_z) - 4\hat{a}_z$

$\boxed{\rho = 20\,\hat{a}_x}$

$|\rho| = \sqrt{20^2} = 20$

$\therefore \hat{a}_\rho = \frac{\vec{\rho}}{|\rho|} = \frac{20\hat{a}_x}{20}$

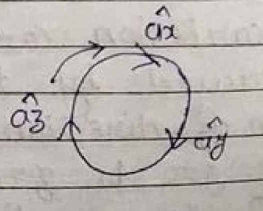


$\boxed{\hat{a}_\rho = \hat{a}_x}$

$\therefore \hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho$

$= \hat{a}_z \times a_x$

$= \hat{a}_y$



$\therefore H = \frac{15\text{ Amp}}{2\pi\,20}\,\hat{a}_y = 0.12\hat{a}_y$ Amp/m.

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$Q(2,-4,4)$

$\vec{\rho} = d - s$

$= (2\hat{a_x} + 4\hat{a_y} + \cancel{4\hat{a_z}}) - \cancel{4\hat{a_z}}$

$\therefore \vec{\rho} = 2\hat{a_x} - 4\hat{a_y}$

$S(0,0,4)$ $z$, $x=0$, $y=0$

$\therefore |\rho| = \sqrt{4+16} = \sqrt{20} = \sqrt{5}\sqrt{2}$

$\therefore \hat{a_\rho} = \frac{2\hat{a_x} - 4\hat{a_y}}{2\sqrt{5}} = \frac{\cancel{2}(2\hat{a_x} + 2\hat{a_y})}{\cancel{2}\sqrt{5}}$

$\therefore \hat{a_\phi} = \hat{a_I} \times \hat{a_\rho}$

$= \hat{a_z} \times \frac{2\hat{a_x} - 2\hat{a_y}}{\sqrt{5}} = \frac{\hat{a_y} + 2\hat{a_x}}{\sqrt{5}}$

$\therefore H = \frac{15}{2 \times 3.14 \times 2\sqrt{5} \times \sqrt{5}} (2\hat{a_x} + 2\hat{a_y})$

$\therefore H = 0.238 (2\hat{a_x} + 2\hat{a_y})$

$\times \therefore$ ~~$H = 0.238\,\hat{a_x} - 0.477\,\hat{a_y}$ Amp/m.~~

$\boxed{H = 0.476\,\hat{a_x} + 0.238\,\hat{a_y} \text{ Amp/m}}$

Ex.2 Find the vector magnetic field intensity in cartesion co-ordinate at pt. P(1.5, 2, 3) caused by the current felament of 24 Amps in $\hat{a_z}$ direction on z-axis & extending from
(a) $z=0$ to $z=6$ (b) $z=6$ to $z=\infty$
(c) $z=-\infty$ to $z=\infty$

Soln:- (c) $H = \frac{I}{2\pi\rho}\hat{a_\phi}$

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Given: $z=-\infty$ to $z=+\infty$

$P(1.5, 2, 3)$

$\vec{\rho} = d - s$

$= (1.5\hat{a_x} + 2\hat{a_y} + \cancel{3\hat{a_z}}) - \cancel{3\hat{a_z}}$

$S(0,0,3)$ $z$, $x=0$, $y=0$

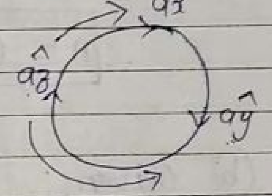
$\vec{\rho} = 1.5\hat{a_x} + 2\hat{a_y}$

$|\rho| = \sqrt{(1.5)^2 + (2)^2}$

$= 2.5$

$\hat{a_\rho} = \frac{\vec{\rho}}{|\rho|} = \frac{1.5\hat{a_x} + 2\hat{a_y}}{2.5}$

$H = \frac{24 \text{ amp}}{2 \times 3.14 \times 2.5 \times 2.5} (1.5\hat{a_y} - 2\hat{a_x})$



$\hat{a_\phi} = \hat{a_I} \times \hat{a_\rho}$

$= \hat{a_z} \times \left(\frac{1.5\hat{a_x} + 2\hat{a_y}}{2.5}\right)$

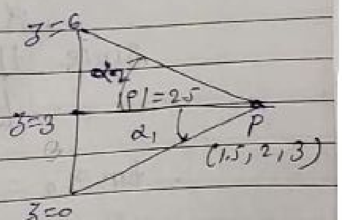
$\hat{a_\phi} = \frac{1}{2.5}(1.5\hat{a_y} - 2\hat{a_x})$

$H = 0.611 (1.5\hat{a_y} - 2\hat{a_x})$

$\boxed{H = 0.917\,\hat{a_y} - 1.22\,\hat{a_x}}$ Amp/m.

(a) Given: $z=0$ to $z=6$.

$H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a_\phi}$



$\tan\alpha_1 = \frac{\text{opp.}}{\text{Adj}} = -\frac{3}{2.5}$

$\therefore \alpha_1 = -\tan^{-1}\left(\frac{3}{2.5}\right)$

$\therefore \alpha_1 = -50.19°$

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$\therefore \tan\alpha_2 = 3/2.5$

$\therefore \alpha_2 = \tan^{-1}(3/2.5)$

$\therefore \alpha_2 = 50.19°$

$\therefore$ Since $\alpha_2$ is above the reference.

$\therefore \boxed{\alpha_2 = 50.19°}$

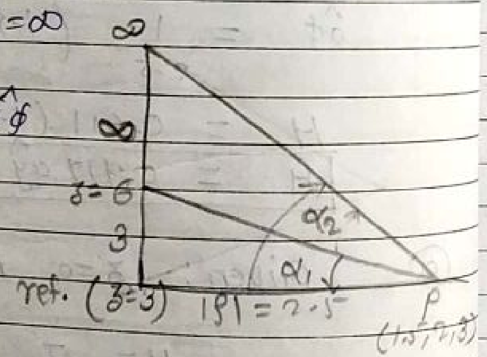
$$\therefore H = \frac{24 \times (\sin(50.19°) - \sin(-50.19°))}{4 \times 3.14 \times 2.5 \times 2.5}(1.5\hat{a_y} - 2\hat{a_x})$$

$\therefore H = 0.305\ (0.768 + 0.768)(1.5\hat{a_y} - 2\hat{a_x})$

$\therefore H = 0.468\ (1.5\hat{a_y} - 2\hat{a_x})$

$\therefore \boxed{H = 0.702\,\hat{a_y} - 0.936\,\hat{a_x}}$ Amp/m

(b) Given : $z = 6$ to $z = \infty$




$$H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a_\phi}$$

$\therefore \tan\alpha_1 = \frac{Opp}{Adj} = \frac{3}{2.5}$

$\therefore \alpha_1 = \tan^{-1}(3/2.5)$

$\therefore \alpha_1 = 50.19°$

Since the $\alpha_1$ lies above the reference line.

$\therefore \boxed{\alpha_1 = 50.19°}$

$\therefore \tan\alpha_2 = \left(\frac{\infty}{2.5}\right) = \tan^{-1}(\infty) = 90°.$

$\therefore$ Since the $\alpha_2$ lies above the reference line

$\therefore \boxed{\alpha_2 = 90°}$

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$$\therefore H = \frac{24\ (\sin(90°) - \sin(50.19°))}{4\pi \times 2.5 \times 2.5}(1.5\hat{a_y} - 2\hat{a_x})$$

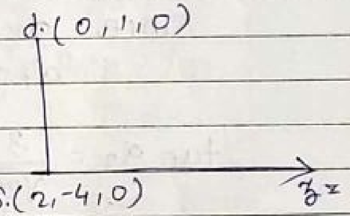
$\therefore H = 0.305\ (1 - 0.768)(1.5\hat{a_y} - 2\hat{a_x})$

$\therefore H = 0.070\ (1.5\hat{a_y} - 2\hat{a_x})$

$\therefore \boxed{H = 0.105\,\hat{a_y} - 0.14\,\hat{a_x}}$ Amp/m.

Ex.3 A current of 0.4 Amps. in $\hat{a_z}$ direction in free space is in a felament ||el to z-axis & is passing through the point (2, −4, 0) Find H at (0, 1, 0) if the felament lies in the interval (a) $-\infty < z < \infty$ (b) $-3 < z < 3$ (c) $0 < z < \infty$

Soln:- (a) Given $-\infty < z < \infty$



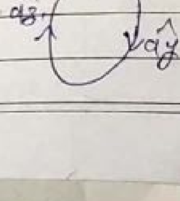

$$\therefore H = \frac{I}{2\pi\rho}\hat{a_\phi}$$

$\therefore \vec{\rho} = d - s = (\hat{a_y}) - (2\hat{a_x} - 4\hat{a_y} + 0\hat{a_z})$

$\therefore \vec{\rho} = 5\hat{a_y} - 2\hat{a_x}$ or $\vec{\rho} - 2\hat{a_x} + 5\hat{a_y}$

$\therefore |\rho| = \sqrt{4 + 25} = \sqrt{29}$

$$\therefore \hat{a_\rho} = \frac{-2\hat{a_x} + 5\hat{a_y}}{\sqrt{29}}$$




$$\therefore \hat{a_\phi} = \hat{a_z} \times \hat{a_\rho} = \hat{a_z} \times \frac{-2\hat{a_x} + 5\hat{a_y}}{\sqrt{29}}$$

$$\therefore \hat{a_\phi} = \frac{1}{\sqrt{29}}(-2\hat{a_y} - 5\hat{a_x})$$

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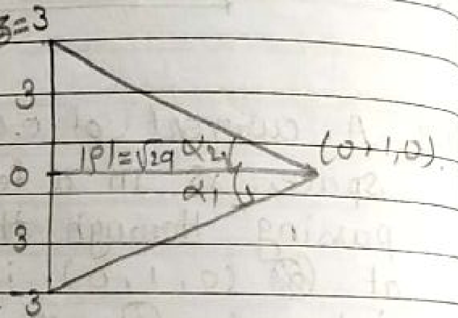
$\therefore H = \dfrac{0.4}{2\times3.14\times\sqrt{29}\times\sqrt{29}}(-2\hat{a_y} - 5\hat{a_x})$

$\therefore H = 2.196\times10^{-3}(-2\hat{a_y} - 5\hat{a_x})$

$\therefore \boxed{H = (-4.392\,\hat{a_y} - 10.98\,\hat{a_x})\times10^{-3}}$ Amp/m.

) Given:- $-3 < z < 3$.




$\therefore H = \dfrac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a_\phi}$

$\therefore \tan\alpha_1 = \dfrac{3}{\sqrt{29}} = \tan^{-1}\left(\dfrac{3}{\sqrt{29}}\right)$

$\therefore \alpha_1 = 29.12^\circ$

Since $\alpha_1$ lies below the reference line:

$\alpha_1 = -29.12^\circ$

$\tan\alpha_2 = \dfrac{3}{\sqrt{29}}$ $\therefore \tan^{-1}(3/\sqrt{29}) = \alpha_2$

$\therefore \alpha_2 = 29.12^\circ$

Since $\alpha_2$ lies above the reference line.

$\therefore \alpha_2 = 29.12^\circ$

$\therefore H = \dfrac{0.4(\sin(29.12) - \sin(-29.12))(-2\hat{a_y} - 5\hat{a_x})}{4\times3.14\times\sqrt{29}\times\sqrt{29}}$

$\therefore H = 1.098\times10^{-3}(0.486+0.486)(-2\hat{a_y} - 5\hat{a_x})$

$= 1.067\times10^{-3}(-2\hat{a_y} - 5\hat{a_x})$

$\boxed{H = (-2.134\,\hat{a_y} - 5.335\,\hat{a_x})}$ mAmp/m.

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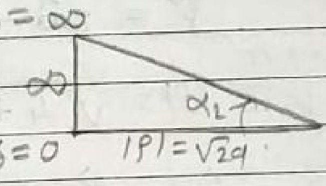
(c) Given:- $0 < z < \infty$




$H = \dfrac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a_\phi}$

$\therefore \tan\alpha_1 = \dfrac{0}{\sqrt{29}}$

$\therefore \tan^{-1}(0) = \alpha_1$ $\therefore \alpha_1 = 0$

$\therefore \tan\alpha_2 = \dfrac{\infty}{\sqrt{29}}$ $\therefore \alpha_2 = \tan^{-1}(\infty) = 90^\circ$

Since the $\alpha_2$ lies above the reference line.

$\alpha_2 = 90^\circ$.

$\therefore H = \dfrac{0.4(\sin(90^\circ) - \sin(0))(-2\hat{a_y} - 5\hat{a_x})}{4\times3.14\times\sqrt{29}\times\sqrt{29}}$

$\therefore H = 1.098\times10^{-3}(1-0)(-2\hat{a_y} - 5\hat{a_y})$

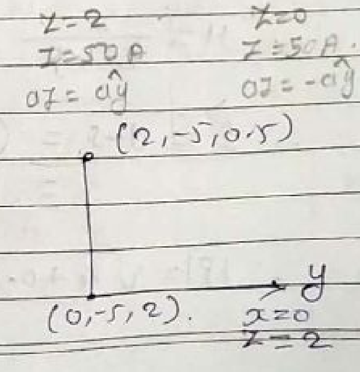
$\therefore \boxed{H = (-2.20\,\hat{a_y} - 5.49\,\hat{a_y})}$ mAmp/m.

Ex.4. 2 infinite current felaments lie in $x=0$ plane. One at $z=2$ carries 50 Amps in $\hat{a_y}$ direction, while the other at $z=0$ carries 50 Amps. in $-\hat{a_y}$ directn. Find $H$ at $(2, -5, 0.5)$.

Soln:- (1) $z = 2$

$I = 50\,A$

$a_I = \hat{a_y}$




$\therefore H = \dfrac{I}{2\pi\rho}\,\hat{a_\phi}$

$\therefore \vec{\rho} = d - s = (2\hat{a_x} - 5\hat{a_y} + 0.5\hat{a_z})$

$- (-5\hat{a_y} + 2\hat{a_z})$

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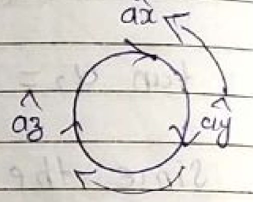
$\therefore \vec{\rho} = 2\hat{a}_x - 1.5\hat{a}_z$

$\therefore |\rho| = \sqrt{4 + 2.25} = 2.5$

$\therefore \hat{a}_\rho = \dfrac{\vec{\rho}}{|\rho|} = \dfrac{2\hat{a}_x - 1.5\hat{a}_z}{2.5}$




$\therefore \hat{a}_\phi = a_I \times \hat{a}_\rho$

$= \hat{a}_y \times \dfrac{2\hat{a}_x - 1.5\hat{a}_z}{2.5}$

$\hat{a}_\phi = \dfrac{-2\hat{a}_z - 1.5\hat{a}_x}{2.5}$

$\therefore H = \dfrac{50 \; (-2\hat{a}_z - 1.5\hat{a}_x)}{2\times 3.14 \times 2.5 \times 2.5}$

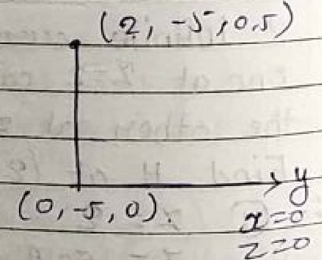
$\therefore H = 1.273\;(-2\hat{a}_z - 1.5\hat{a}_x)$

$\therefore \boxed{H_1 = -2.547\,\hat{a}_z - 1.909\,\hat{a}_x}$ A/m

(b) $z = 0$

$I = 50$

$a_I = -\hat{a}_y$




$\therefore H = \dfrac{I}{2\pi\rho}\,\hat{a}_\phi$

$\therefore \vec{\rho} = d - s = (2\hat{a}_x - \cancel{5\hat{a}_y} + 0.5\hat{a}_z) + \cancel{5\hat{a}_y}$

$= 2\hat{a}_x + 0.5\hat{a}_z$

$\therefore |\rho| = \sqrt{4 + 0.25} = 2.061$ or $\sqrt{4.25}$

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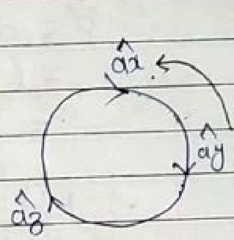
$\hat{a}_\rho = \dfrac{\vec{\rho}}{|\rho|} = \dfrac{2\hat{a}_x + 0.5\hat{a}_z}{\sqrt{4.25}}$




$\therefore \hat{a}_\phi = a_I \times \hat{a}_\rho$

$= -\hat{a}_y \times \dfrac{2\hat{a}_x + 0.5\hat{a}_z}{\sqrt{4.25}}$

$\hat{a}_\phi = \dfrac{2\hat{a}_z - 0.5\hat{a}_x}{\sqrt{4.25}}$

$\therefore H = \dfrac{50 \; (2\hat{a}_z - 0.5\hat{a}_x)}{2\times 3.14 \times \sqrt{4.25} \times \sqrt{4.25}}$

$\therefore H = 1.873\;(2\hat{a}_z - 0.5\hat{a}_x)$

$\boxed{H_2 = 3.746\,\hat{a}_z - 0.936\,\hat{a}_x}$ A/m.

$\therefore H = H_1 + H_2 = -2.547\,\hat{a}_z - 1.909\,\hat{a}_x + 3.746\,\hat{a}_z - 0.936\,\hat{a}_x$

$\boxed{H_T = 1.199\,\hat{a}_z - 2.845\,\hat{a}_x}$ A/m.

Assg.

Ex.5) A felamentary current of 6 Amps. flows inwards along +ve x-axis from $x = \infty$ to $x = 0$. and then outwards along +ve y-axis from $y = 0$ to $y = \infty$ find H at (1) A(1, 1, 0) (2) B (0, 0, 1)

Soln:-

| (1) x | (2) y |
|---|---|
| $x = \infty$ to $x = 0$ | $y = 0$ to $y = \infty$ |
| $a_I = -\hat{a}_x$ | $a_I = \hat{a}_y$ |

$I = 6$ AMP.

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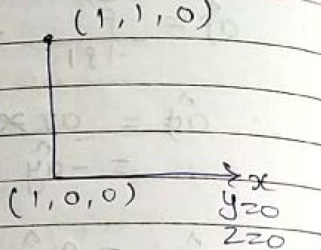
$\therefore \vec{\rho} = d - s$

$= (\cancel{\hat{a_x}} + \hat{a_y}) - \cancel{\hat{a_x}}$

$= \hat{a_y}$



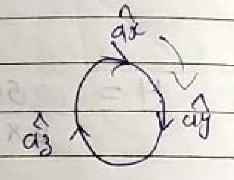

$\therefore |\rho| = \sqrt{1} = 1$

$\therefore \hat{a_\rho} = \hat{a_y}/1 = \hat{a_y}$

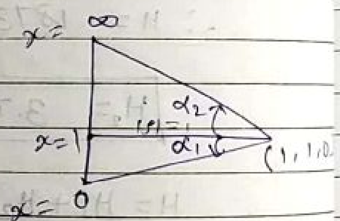
$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho}$

$= -\hat{a_x} \times \hat{a_y}$

$\hat{a_\phi} = -\hat{a_z}$




$\therefore H = \dfrac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a_\phi}$




$\therefore \alpha_1 = \tan^{-1}\left(\frac{1}{1}\right)$

$\therefore \alpha_1 = -45^\circ$

$\alpha_2 = \tan^{-1}\left(\frac{\infty}{1}\right) = 90^\circ$

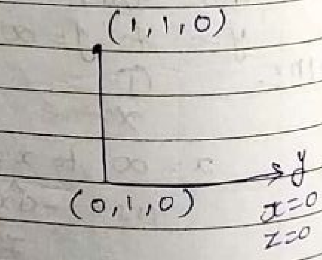
$\therefore H = \dfrac{6}{4 \times 3.14 \times 1}(\sin(90^\circ) - \sin(-45^\circ))(-\hat{a_z})$

$\therefore H = 0.477\,(1 + 0.707)(-\hat{a_z})$

$\therefore \boxed{H_1 = -0.814\,\hat{a_z}}$

(2) $\vec{\rho} = \hat{a_x} + \cancel{\hat{a_y}} - \cancel{\hat{a_y}}$

$= \hat{a_x}$




$\therefore |\rho| = \sqrt{1} = 1$

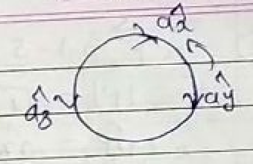
$\therefore \hat{a_\rho} = \hat{a_x}$

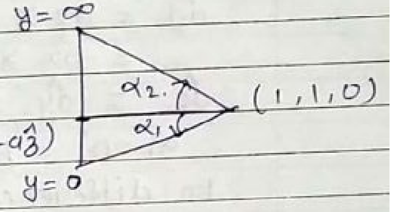


$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho}$

$= \hat{a_y} \times \hat{a_x}$

$\hat{a_\phi} = -\hat{a_z}$




$\therefore \alpha_1 = -45^\circ$

$\alpha_2 = 90^\circ$

$\therefore H = \dfrac{6}{4 \times 3.14 \times 1}(\sin(90^\circ) - \sin(-45^\circ))(-\hat{a_z})$

$\therefore H_2 = (0.477)(1.707)(-\hat{a_z})$

$\boxed{H_2 = -0.814\,\hat{a_z}}$

$H_T = H_1 + H_2 = 2 - (0.814\,\hat{a_z})$

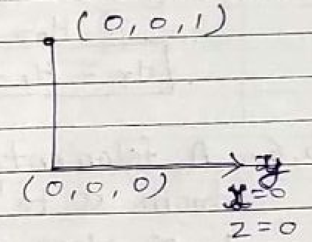
$\therefore \boxed{H_T = -1.628\,\hat{a_z}}$ A/m.

B) $\therefore \vec{\rho} = d - s$

$= \hat{a_z}$

$|\rho| = \sqrt{1} = 1$ $\qquad a_I = \hat{a_y}$

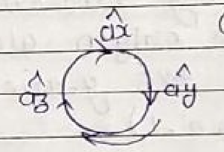
$\hat{a_\rho} = \hat{a_z}$




$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho}$

$= \hat{a_y} \times \hat{a_z}$

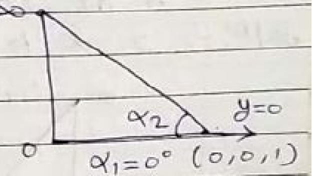
$= \hat{a_x}$




$\therefore \tan\alpha_2 = \left(\frac{\infty}{1}\right)$

$\therefore \alpha_2 = \tan^{-1}(\infty)$

$\therefore \alpha_2 = 90^\circ$




$\therefore H = \dfrac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\,a_\phi$

$\therefore H = \dfrac{6\ \hat{a_x}}{4 \times 3.14 \times 1} = 0.477\,\hat{a_x}$

$\therefore H_1 = 0.477\,\hat{a_x}$

② $\therefore \vec{\rho} = d - s = \hat{a_z}$

$\therefore |\rho| = \sqrt{1} = 1$

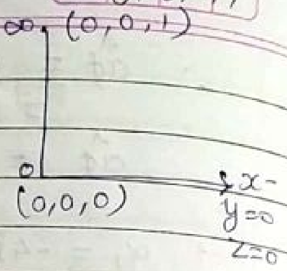
$\therefore \hat{a_\rho} = \hat{a_z}$

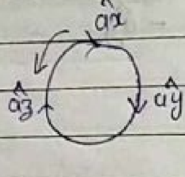
$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho}$

$= -\hat{a_x} \times \hat{a_z}$

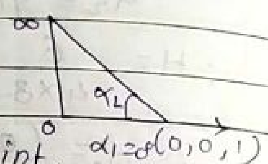
$\hat{a_\phi} = \hat{a_y}$

$a_I = -\hat{a_x}$




$\therefore \alpha_1 = 0^\circ$ because their is no difference between the refference line & the $x=0$ point.




$\tan \alpha_2 = \left(\frac{\infty}{1}\right)$ $\therefore \tan^{-1}(\infty) = \alpha_2$

$\therefore \alpha_2 = 90^\circ$

$\therefore H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a_\phi}$

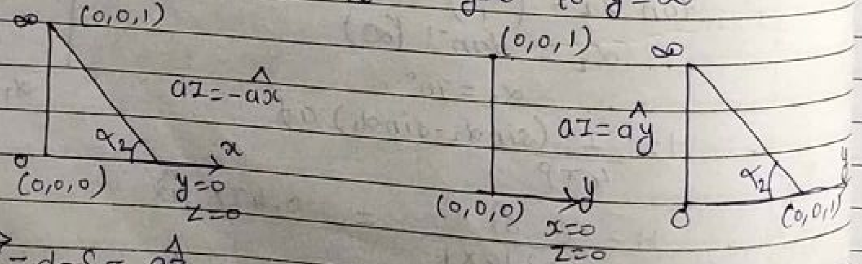
$\therefore H = \frac{6}{4 \times 3.14}(\sin 90^\circ - \sin 0^\circ)\hat{a_y}$

$\therefore H_2 = 0.477\ \hat{a_y}$

$\therefore \boxed{H_T = H_1 + H_2 = 0.477\ \hat{a_x} + 0.477\ \hat{a_y}}$ A/m.

Q.6. A filamentry current of 10 amp. is directed in from $\infty$ to origin along the +ve x-axis & then back to $\infty$ along +ve y-axis. Use Biot Savart law to find H at (0,0,1).

Sol$^n$:- $x=\infty$ to $x=0$ & $y=0$ to $y=\infty$




$\therefore \vec{\rho} = d - s = \hat{a_z}$

$|\rho| = \sqrt{1} = 1$

$\vec{\rho} = d - s = \hat{a_z}$

$|\rho| = \sqrt{1} = 1$



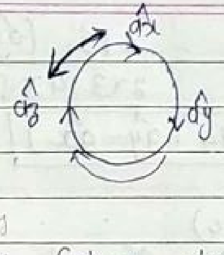
$dL = \int \frac{I dL \times \hat{a_r}}{4\pi r^2}$

$\therefore \hat{a_\rho} = \hat{a_z}$

$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho}$

$\therefore \hat{a_\phi} = -\hat{a_x} \times \hat{a_z}$

$\therefore \hat{a_\phi} = +\hat{a_y}$




$\hat{a_\rho} = \hat{a_z}$

$\hat{a_\phi} = a_I \times \hat{a_\rho}$

$\hat{a_\phi} = \hat{a_y} \times \hat{a_z}$

$\hat{a_\phi} = \hat{a_x}$

$\therefore$ ~~$H_2$~~ $H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a_\phi}$

$\therefore$ ~~$H_2$ to~~ $\tan\alpha_2 = \left(\frac{\infty}{1}\right)$ $\qquad \tan\alpha_2 = (\infty/1)$

$\therefore \tan^{-1}(\infty) = \alpha_2$ $\qquad \therefore \alpha_2 = \tan^{-1}(\infty)$

$\therefore \alpha_2 = 90^\circ$ $\qquad \alpha_2 = 90^\circ$

& $\alpha_1 = 0^\circ$ $\qquad$ & $\alpha_1 = 0^\circ$

$\therefore H = \frac{10(\sin 90^\circ - \sin 0^\circ)}{4 \times 3.14 \times 1}\hat{a_y}$ $\qquad H = \frac{10(\sin 90^\circ - \sin 0^\circ)}{4 \times 3.14 \times 1}\hat{a_x}$

$\therefore H_1 = 0.796\ \hat{a_y}$ A/m $\qquad H_2 = 0.796\ \hat{a_x}$ A/m.

$\therefore \boxed{H_T = H_1 + H_2 = 0.796\ \hat{a_y} + 0.796\ \hat{a_x}}$ A/m

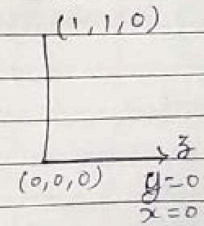
Q.7. A current felament of 15 amp in $\hat{a_z}$ direction lies along the entire z-axis Find H at P(1,1,0) & Q(2,-4,4).

Sol$^n$:- H at P(1,1,0) $\quad I = 15$ amp $\quad a_I = \hat{a_z}$

$\vec{\rho} = d - s = \hat{a_x} + \hat{a_y}$

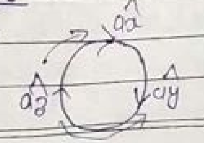
$|\rho| = \sqrt{2}$

$\therefore \hat{a_\rho} = \frac{\hat{a_x} + \hat{a_y}}{\sqrt{2}}$




$\therefore \hat{a_\phi} = a_I \times \hat{a_\rho} = \hat{a_z} \times \frac{\hat{a_x} + \hat{a_y}}{\sqrt{2}}$

$\therefore \hat{a_\phi} = \frac{1}{\sqrt{2}}(\hat{a_y} - \hat{a_x})$

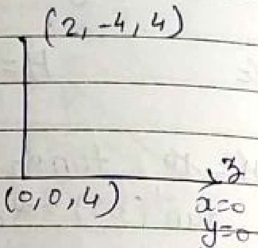


Date. 31-01-17

$\therefore H = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{15}{2\times3.14\times\sqrt{2}\times\sqrt{2}}(\hat{a}_y - \hat{a}_x)$

$\therefore \boxed{H_1 = 1.194(\hat{a}_y - \hat{a}_x)}$ A/m.

② H at Q (2, -4, 4)



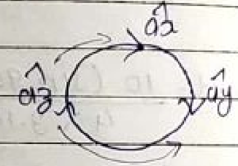
$\vec{\rho} = d - s = 2\hat{a}_x - 4\hat{a}_y + 4\hat{a}_z - 4\hat{a}_z$

$\therefore \vec{\rho} = 2a_x - 4\hat{a}_y$

$|\rho| = \sqrt{4+16} = \sqrt{20}$

$\therefore \hat{a}_\rho = 2\hat{a}_x - 4\hat{a}_y / \sqrt{20}$

$a_I = \hat{a}_z$



$\therefore \hat{a}_\phi = a_I \times \hat{a}_\rho = \hat{a}_z \times \frac{2\hat{a}_x - 4\hat{a}_y}{\sqrt{20}}$

$\hat{a}_\phi = \frac{2\hat{a}_y + 4\hat{a}_x}{\sqrt{20}}$

$\therefore H = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{15(2\hat{a}_y + 4\hat{a}_x)}{2\times3.14\times\sqrt{20}\times\sqrt{20}}$

$\therefore H = 0.1194(2\hat{a}_y + 4\hat{a}_x)$

$\therefore \boxed{H = 0.238\hat{a}_y + 0.477\hat{a}_x}$ A/m.

Date. 04-02-17

Compare

| Magnetic field Intensity & flux density. | Electric field Intensity & flux density. |
|---|---|
| H = Magnetic field Intensity. | E = Electric field Intensity. |
| B = Magnetic flux density. | D = Electric Flux density. |
| $\phi$ = Flux. | $\psi$ = flux. |
| i) $B = \mu H$ <br> $\mu = \mu_0 \mu_r$ <br> $\mu_r = 1$ <br> $\therefore \mu = \mu_0$ <br> where $\mu_0 = 4\pi\times10^{-7}$ wb/m² | i) $D = \epsilon E$ <br> $\epsilon = \epsilon_0 \epsilon_r$ <br> $\epsilon_r = 1$ <br> $\epsilon = \epsilon_0$ <br> $\epsilon_0 = 8.854\times10^{-12}$ F/m. |
| ii) $\phi = \int B \cdot ds$. | ii) $\psi = Q = \int D \cdot ds$. |
| iii) $\nabla \cdot B = 0$ | iii) $\nabla \cdot D = \rho_v$ |
| iv) $\int B \cdot ds = 0$ | iv) $\int D \cdot ds = \rho_v$. |

* Magnetic flux and Magnetic flux density :-

The magnetic flux density is represented by

$B = \mu H$.

where, $\mu = \mu_0 \mu_r$ → permeability.

$\mu_r = 1$ for ~~fee~~ free space.

$B = \mu_0 H$ (for free space).

B is measured in wb/m² or Tesla (T).

The constant $\mu_0$ is not dimensionless & has the defined value for free space in Henry's per ~~unit~~ meter (H/m).

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$\mu_0 = 4\pi \times 10^{-7}\, H/m$

Magnetic flux density is.

$B = \mu_0 H$ (in free space).

where, H is magnetic field intensity

Similarly,

Electric flux density is

$D = \epsilon E$

where, $\epsilon = \epsilon_0 \epsilon_r$ $\quad \epsilon_r = 1$

$\epsilon = \epsilon_0$ [permitivity of free space]

$\therefore D = \epsilon_0 E$

Magnetic flux represented by $\phi$ & is defined as flux passing through any designated Area.

$$\phi = \int_S B.ds \ wb.$$

Similarly,

Electric flux represented by $\psi$ is defined by Gauss Law which states that the total flux passing through any closed surface is equal to the charge enclosed.

$$\psi = \oint_S D.ds = Q.$$

charge Q is the ~~surface~~ source of lines of electric flux and these lines begin and terminate on +ve and -ve charge respectively.

No such source has been discovered for the lines of magnetic flux.

The magnetic flux lines are closed & do not terminate on magnetic charge. The Gauss law for magnetic field is $\oint B.ds = 0$.

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Application of divergence theorem,

$\nabla . B = 0.$

08-02-17 MMSMP

### * Scalar Magnetic Potential *

- when any scaler quantity is multiply by the ($\nabla$) with negative sign is called as the "gradient". i.e. $\underset{E}{-\nabla V_m} =$ gradient.

- when any vector quantity is crossed by the ($\nabla$) then it is called as the "Curle". i.e. $\nabla \times H = J$ and J is the current density over the surface.

In Scalar Magnetic Potential, ($V_m$)

① Curle of a gradient $= 0$

② Laplacion Operator $= 0$. $\quad (\nabla^2 V_m) = 0$.

①. $H = -\nabla V_m$ --- (Gradient)

$\therefore \nabla \times H = J$ --- (curl of gradient)

$\therefore \nabla \times (-\nabla V_m) = J$

$\therefore 0 = J$ i.e. $\boxed{J = 0}$

② In Magnetic field

$\nabla . B = 0.$

$B = \mu H$

$\mu = \mu_0 \mu_r H$

But $\mu_r = 1$ in free space.

$\therefore B = \mu_0 H$

$\therefore \nabla . \mu_0 H = 0.$

$\therefore \mu_0 (\nabla . H) = 0.$

but $H = -\nabla V_m$ --- ①.

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$\therefore \mu_0 (\nabla \cdot (-\nabla V_m)) = 0$.

$\therefore \mu_0 (-\nabla^2 V_m) = 0$.

$-\nabla^2 V_m = 0$.

$\therefore \boxed{\nabla^2 V_m = 0}$

Hence proved.

MMIMP

## * Vector Magnetic Potential *

1. Curl of curl $\neq 0$
2. Divergence $= 0$.

let Vector Magnetic Potential $= A$.

$\therefore B = \nabla \times A$

But $B = \mu H$

$\mu = \mu_0 \mu_r H$

but $\mu_r = 1$ in free space.

$\therefore \mu = \mu_0 H$

$\therefore \mu_0 H = \nabla \times A$

$\therefore H = \frac{1}{\mu_0} (\nabla \times A)$.

$\therefore$ Taking curl on both side, we get.

$\nabla \times H = \frac{1}{\mu_0} (\nabla \times (\nabla \times A))$

$\therefore \boxed{J = \frac{1}{\mu_0} (\nabla \times (\nabla \times A))}$

Hence proved that curl of a curl is not equals to zero.

We know that, $\nabla \cdot B = 0$.

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## * Ampere Circuital Law *

It states that, "The line integral of H about any closed path is exactly equal to the direct current enclosed by that path."

$$\boxed{\oint_{line} H \cdot dL = I}$$

$H = H_\phi \hat{a_\phi}$ and $dL = \rho \, d\phi \, \hat{a_\phi}$

$\therefore \oint H_\phi \hat{a_\phi} \cdot \rho \, d\phi \, \hat{a_\phi} = I$.

$\therefore H_\phi \rho \int_0^{2\pi} d\phi = I$.

$\therefore H_\phi \rho [\phi]_0^{2\pi} = I$.

$\therefore H_\phi \rho [2\pi] = I$.

$\therefore H_\phi = \frac{I}{2\pi \rho}$

$\therefore \boxed{H = \frac{I}{2\pi\rho} \hat{a_\phi}}$

Hence proved.

$I = \frac{dQ}{dt}$ --- (1) It is rate of change of charge w.r.t. time.

$I = \int H \cdot dL$ --- (2) By Ampere circuital law

$I = \int J \cdot ds$ --- (3) By defination.

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## * Continuity Equation * $\nabla \cdot J = -\frac{\partial}{\partial t} \rho_v$.

Q. Consider a closed surface enclosed in a total charge of 'Q' columbs.

Let a current of 'I' amp's. flow out of the closed surface. The current is defined as "The rate of flow of charge".

Therefore the current that flows out of the closed surface results in decrease of charge inside the closed surface.

A current that results in decreased in stored charge is called as "-ve current".

$$I = -\frac{dQ}{dt} \quad \text{---} \; (1)$$

As the total current 'I' crosses the closed surface inoutward direction their exists current density 'J' amp/m$^2$ at each pt. on the closed surface.

$$I = \int J \cdot ds \quad \text{----} \; (2).$$

$\therefore$ from eq$^n$ (1) & (2) we get,

$$-\frac{dQ}{dt} = \int J \cdot ds. \quad \text{---} \; (3).$$

The total charge lies in the volume by the closed surface is given by,

$$Q = \int \rho_v \, dv \quad \text{---} \; (4).$$

$\therefore$ eq$^n$ (3) becomes.

$$\int J \cdot ds = -\frac{d}{dt} \int \rho_v \, dv$$

$$\therefore \boxed{\int J \cdot ds = -\int \frac{d}{dt} \rho_v \, dv.} \quad \text{--} \; (4).$$

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There is the integral form of continuity equation.

Formula of divergence is given by,

$$\nabla \cdot D = \rho_v.$$

also $\int D \cdot ds = \int \rho_v \, dv$.

$\therefore$ Put $\rho_v$ value in above eq$^n$

$$\therefore \int D \cdot ds = \int (\nabla \cdot D) \, dv \quad \text{---} \; (5).$$

$\therefore$ from eq$^n$ (5) eq$^n$ (4) becomes.

$$\int (\nabla \cdot J) \, dv = -\int \frac{\partial}{\partial t} \rho_v \, dv$$

$$\therefore \boxed{\nabla \cdot J = -\partial/\partial t \; \rho_v}$$

Hence proved.