Compensation

Before introducing various compensation **in control system** in detail, it is very essential to know the uses of compensating networks in the control system. The important uses of the compensating networks are written below.

Necessary of Compensation

1. In order to obtain the desired performance of the system, we use compensating networks. Compensating networks are applied to the system in the form of feed forward path gain adjustment.
2. Compensate a unstable system to make it stable.
3. A compensating network is used to minimize overshoot.
4. These compensating networks increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system.
5. Compensating networks also introduces poles and zeros in the system thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system change.

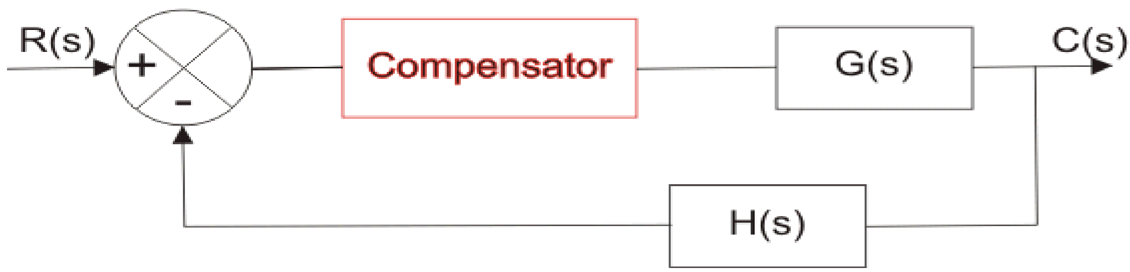
Methods of Compensation

1. Connecting compensating circuit between error detector and plants known as **Cascade/series compensation**.
   Cascade compensation is perhaps the most common control system topology. With cascade compensation the error signal is found, and the control signal is developed entirely from the error signal

   In cascade compensation, the system transfer function is

   $$H_s = G_C G(s) / (1 + G_C G(s))$$

   with the salient characteristic that the entire controller characteristic appears in both the numerator and denominator of the transfer function—so anything you do to affect the system poles also affects the system zeros.



Series/Cascade Compensator

2. When a compensator used in a feedback manner called **feedback compensation**.

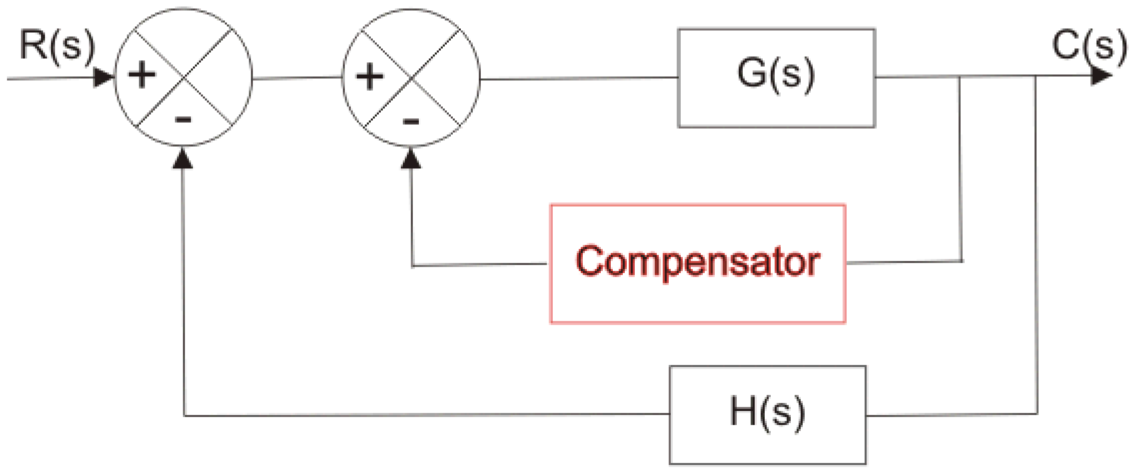
Feedback compensation, shown in Figure below, modifies the plant output in some way before it is compared to the command signal. This is generally done because the property that we wish to control is not the one that we are measuring; for example, if we want to control the velocity of a system with position measurement. Showing a functional block in the feedback path is also a good way to model how the measurement process affects the signal. Whether it is a simple gain block to model an analog-to-digital converter, or something more complex such as a gyroscope or an accelerometer, you can model the behavior of a sensor with a feedback block

The transfer function of a system with feedback control is

$$H_s = G_C\,G(s) / (1 + G_C\,G(s)H(S))$$

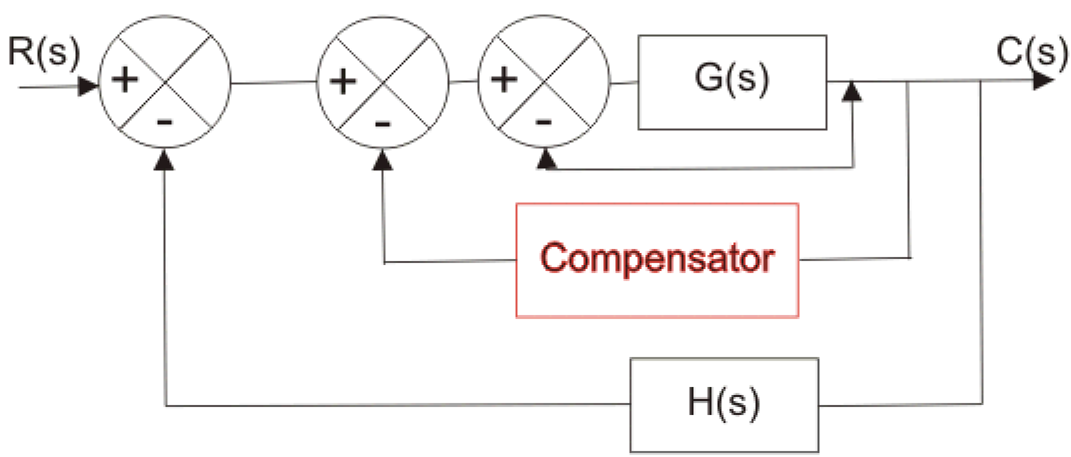
Putting compensation in the feedback path of a system, either explicitly in the form of a filter, or implicitly in the form of a sensor, can have a dramatic effect on the "personality" of the system. Instead of the system attempting to servo to the system output, it will now attempt to servo to the output of the feedback compensation; thus, a rotary system with a gyroscope or tachometer in the feedback path will tend to servo the plant's rate of rotation, while a system with an accelerometer will tend to servo to the plant's acceleration.

Careful use of feedback compensation, particularly by sensor selection, can be very powerful indeed



Feedback Compensator

3. A combination of series and feedback compensator is called **load compensation**.



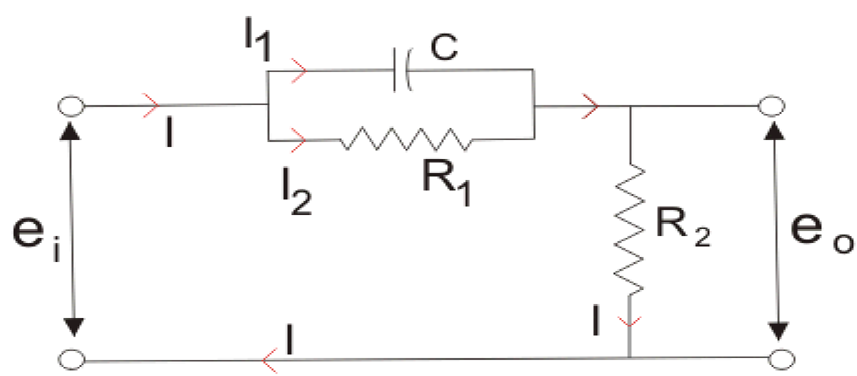
Load Compensator

Now what are compensating networks?

A compensating network is one which makes some adjustments in order to make up for deficiencies in the system. Compensating devices are may be in the form of electrical, mechanical, hydraulic etc. Most electrical compensator are RC filter. The simplest network used for compensator are known as lead, lag network.

Phase Lead Compensation

A system which has one pole and one dominating zero (the zero which is closer to the origin than all over zeros is known as dominating zero.) is known as lead network. If we want to add a dominating zero for **compensation in control system** then we have to select **lead compensation** network. The basic requirement of the phase lead network is that all poles and zeros of the transfer function of the network must lie on (-)ve real axis interlacing each other with a zero located at the origin of nearest origin. Given below is the circuit diagram for the phase **lead compensation** network.



Phase Lead Compensation Network

From above circuit we get,

$$I_1 = C\frac{d}{dt}(e_i - e_o)$$
$$I_2 = \frac{e_i - e_o}{R_1}$$
$$I = I_1 + I_2 = C\frac{d}{dt}(e_i - e_o) + \frac{e_i - e_o}{R_1}$$
$$Again, I = \frac{e_o}{R_2}$$

Equating above expression of I we get,

$$\frac{e_o}{R_2} = C\frac{d}{dt}(e_i - e_o) + \frac{e_i - e_o}{R_1}$$

Now let us determine the transfer function for the given network and the transfer function can be determined by finding the ratio of the output voltage to the input voltage.

So taking Laplace transform of both side of above equations,

$$\frac{1}{R_2}E_o(s) = \frac{1}{R_1}[E_i(s) - E_o(s)] + Cs[E_i(s) - E_o(s)] \quad (neglecting\ initial\ condition)$$
$$\Rightarrow \frac{1}{R_2}E_o(s) + \frac{1}{R_1}E_o(s) + CsE_o(s) = \frac{E_i(s)}{R_1} + CsE_i(s)$$

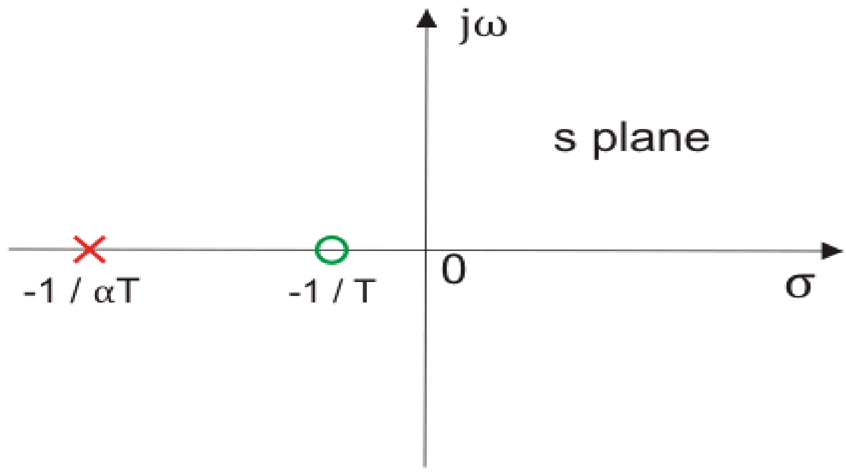
$$\Rightarrow \frac{E_o(s)}{E_i(s)} = \frac{\frac{1+sCR_1}{R_1}}{\frac{R_1+R_2+sR_1R_2C}{R_2R_1}}$$
$$\Rightarrow \frac{E_o(s)}{E_i(s)} = \frac{R_2}{R_1+R_2}\left[\frac{1+sCR_1}{1+\frac{sR_1R_2C}{R_1+R_2}}\right]$$

On substituting the $\alpha = (R_1 + R_2)/ R_2$ and $T = \{(R_1R_2) /(R_1 + R_2)\}$ in the above equation.

Where, T and α are respectively the time constant and attenuation constant, we have

$$Transfer\ function\ ,\ G_{lead}(s) = \frac{E_o(s)}{E_i s} \ = \frac{1}{\alpha}\left[\frac{1+\alpha sT}{1+sT}\right]$$

The above network can be visualized as an amplifier with a gain of $1/\alpha$. Let us draw the pole zero plot for the above transfer function.



Pole Zero Plot of Lead Compensating Network

Clearly we have -1/T (which is a zero of the transfer function) is closer to origin than the -1/(αT) (which is the pole of the transfer function).Thus we can say in the lead compensator zero is more dominating than the pole and because of this lead network introduces positive phase angle to the system when connected in series.
Let us substitute s = jω in the above transfer function and also we have α
Now in order to find put the maximum phase lead occurs at a frequency let us differentiate this phase function and equate it to zero. On solving the above equation we get

$$\alpha = \frac{1-\sin\theta_m}{1+\sin\theta_m}$$

Where, θ_m is the maximum phase lead angle. And the corresponding magnitude of the transfer function at maximum θ_m is 1/a.

Effect of Phase Lead Compensation

1. The velocity constant K_v increases.
2. The slope of the magnitude plot reduces at the gain crossover frequency so that relative stability improves and error decrease due to error is directly proportional to the slope.
3. Phase margin increases.
4. Response become faster.

Advantages of Phase Lead Compensation

Let us discuss some of the advantages of the phase lead compensation-

1. Due to the presence of phase lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lead compensation maximum overshoot of the system decreases.

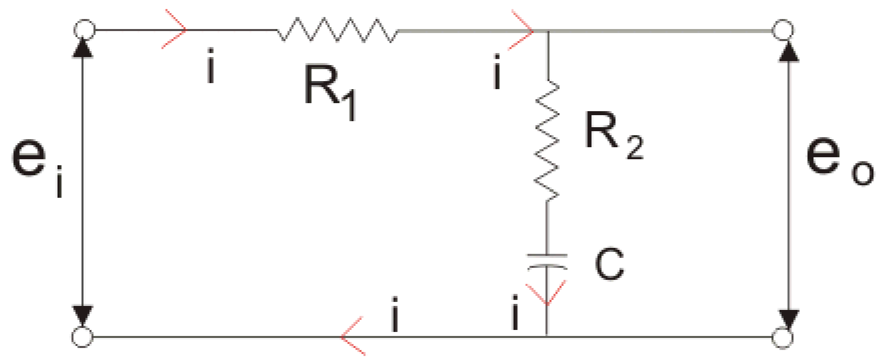
Disadvantages of Phase Lead Compensation

Some of the disadvantages of the phase lead compensation –

1. Steady state error is not improved.

Phase Lag Compensation

A system which has one zero and one dominating pole (the pole which is closer to origin that all other poles is known as dominating pole) is known as lag network. If we want to add a dominating pole for **compensation in control system** then, we have to select a **lag compensation** network.
The basic requirement of the phase lag network is that all poles and zeros of the transfer function of the network must lie in (-)ve real axis interlacing each other with a pole located or on the nearest to the origin.
Given below is the circuit diagram for the phase **lag compensation** network.



Phase Lag Compensating Network

We will have the output at the series combination of the resistor R_2 and the capacitor C.
From the above circuit diagram, we get

$$e_i = iR_1 + iR_2 + \frac{1}{C}\int idt$$

$$e_o = iR_2 + \frac{1}{C}\int idt$$

Now let us determine the transfer function for the given network and the transfer function can be determined by finding the ratio of the output voltage to the input voltage.

Taking Laplace transform of above two equation we get,

$$E_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{Cs} I(s)$$

$$E_o(s) = R_2 I(s) + \frac{1}{Cs} I(s)$$

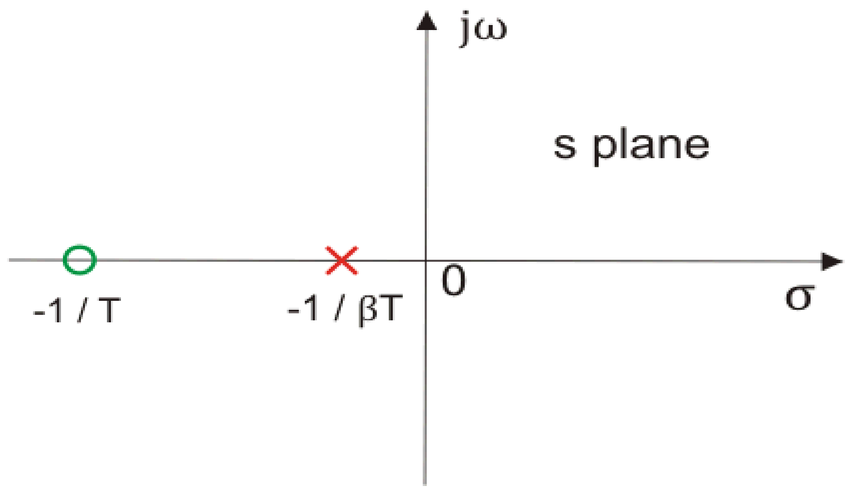
$$Transfer\ function,\ G_{lag}(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}$$

$$\Rightarrow G_{lag}(s) = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1}$$

On substituting the $T = R_2 C\ and\ \beta = \left\{ \frac{(R_2 + R_1)}{R_1} \right\}$ in the above equation (Where, T and β are respectively the time constant and DC gain), we have

$$Transfer\ function,\ G_{lag}(s) = \frac{1 + Ts}{1 + \beta Ts}$$

The above network provides a high frequency gain of 1 / β. Let us draw the pole zero plot for the above transfer function.



Pole Zero Plot of Lag Network

Clearly we have -1/T (which is a zero of the transfer function) is far to origin than the -1 / (βT)(which is the pole of the transfer function). Thus we can say in the lag compensator pole is more dominating than the zero and because of this lag network introduces negative phase angle to the system when connected in series.
Let us substitute s = jω in the above transfer function and also we have a
Now in order to find put the maximum phase lag occurs at a frequency let us differentiate this phase function and equate it to zero. On solving the above equation we get

$$\beta = \frac{1 - \sin\theta_m}{1 + \sin\theta_m}$$

Where, θ_m is the maximum phase lead angle. Remember β is generally chosen to be greater than 10.

Effect of Phase Lag Compensation

1. Gain crossover frequency increases.
2. Bandwidth decreases.
3. Phase margin will be increase.
4. Response will be slower before due to decreasing bandwidth, the rise time and the settling time become larger.

Advantages of Phase Lag Compensation

Let us discuss some of the advantages of phase lag compensation –

1. Phase lag network allows low frequencies and high frequencies are attenuated.
2. Due to the presence of phase lag compensation the steady state accuracy increases.

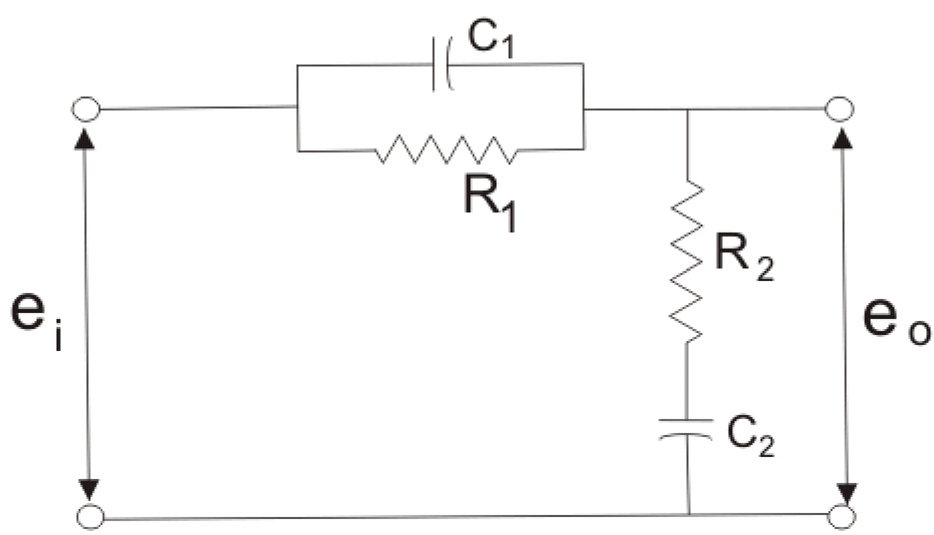
Disadvantages of Phase Lag Compensation

Some of the disadvantages of the phase lag compensation –

1. Due to the presence of phase lag compensation the speed of the system decreases.

Phase Lag Lead Compensation

With single lag or lead compensation may not satisfied design specifications. For an unstable uncompensated system, lead compensation provides fast response but does not provide enough phase margin whereas lag compensation stabilize the system but does not provide enough bandwidth. So we need multiple compensators in cascade.
Given below is the circuit diagram for the phase **lag- lead compensation** network.



Lag Lead Compensating Network

Now let us determine transfer function for the given network and the transfer function can be determined by finding the ratio of the output voltage to the input voltage.

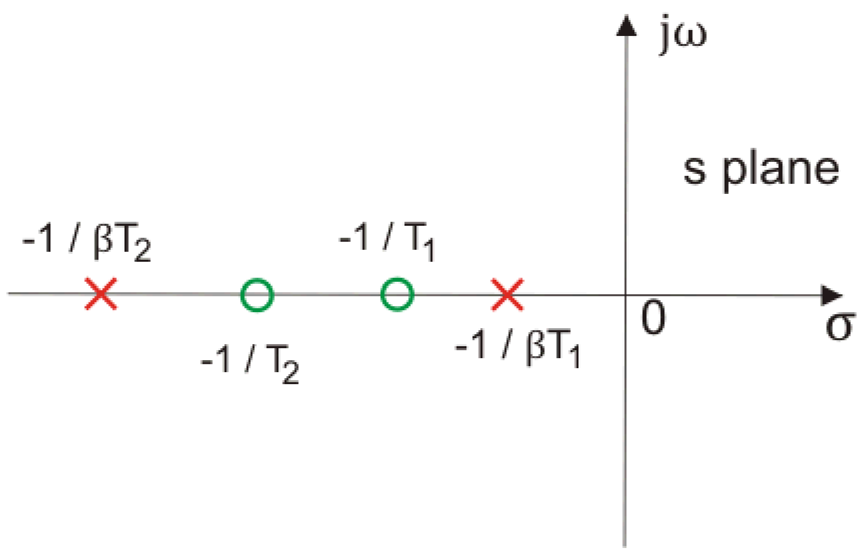
$$Transfer\ function\ , G_{lag-lead}(s) = \frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

$$\Rightarrow G_{lag-lead}(s) = \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$$

On substituting the $\alpha T_1 = R_1C_1$, $R_2C_2 = \beta T_2$, $R_1R_2C_1C_2 = \alpha\beta T_1T_2$ and $T_1T_2 = R_1R_2C_1C_2$ in the above equation (where T_1, T_2 and α, β are respectively the time constants and attenuation constants). We have

$$Transfer\ function\ , G_{lag-lead}(s) = \frac{(1+\alpha T_1 s)(1+\beta T_2 s)}{(1+T_1 s)(1+T_2 s)}$$

Let us draw the pole zero plot for the above transfer function.



Pole Zero Plot Lag Lead Network

Clearly we have -1/T (which is a zero of the transfer function) is far to the origin than the -1/(βT)(which is the pole of the transfer function). Thus we can say in the **lag-lead compensation** pole is more dominating than the zero and because of this lag-lead network may introduces positive phase angle to the system when connected in series.

Advantages of Phase Lag Lead Compensation

Let us discuss some of the advantages of phase lag- lead compensation-

1. Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lag-lead network accuracy is improved.

Comparison of phase lead and lag compensation

	Phase lead compensation	Phase lag compensation
Main Idea	Improve transient performance by using phase lead characteristics	Improve the steady-state performance by using magnitude attenuation at the high-frequency part
Effect	(1) Around ω_c, the absolute value of slope is reduced. Phase margin γ and gain margin GM are increased. (2) Increase the bandwidth (3) With bigger γ, overshoot is reduced. (4) Take no effect on the steady-state performance.	(1) Keep relative stability unchanged, but reduce the steady-state error. (2) Reduce ω_c and then closed-loop bandwidth (3) For specific open-loop gain, γ, GM and resonant peak M_r are all improved due to magnitude attenuation around ω_c
Weakness	(1) Broad bandwidth reduces the filtering for noise. (2) For passive network implementation, need an extra amplifier.	Narrow Bandwidth increase the response time.
Application	(1) Extra phase lead compensation is less than 55^0. (2) Require broad bandwidth and fast response (3) No matter the noise at high-frequency part.	(1) The phase lag of the uncompensated system is fast around ω_c. (2) Bandwidth and transient response are satisfactory. (3) Require attenuation of noise (4) The phase margin can be satisfied at the low frequency.