Tulsiramji Gaikwad-Patil College of Engineering & Technology
Department of Master in Computer Application

Subject Notes
Academic Session: 2018 – 2019

Subject:DMGT

Semester: II

Either

1.(A) Let A, B and C be finite sets, then prove that :
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Solution

We have $A \cup B \cup C = \{a, b, c, d, e, g, h, k, m, n\}$, $A \cap B = \{a, b, e\}$, $A \cap C = \{b, d, e\}$, $B \cap C = \{b, e, g, h\}$, and $A \cap B \cap C = \{b, e\}$, so $|A| = 5$, $|B| = 5$, $|C| = 8$, $|A \cup B \cup C| = 10$, $|A \cap B| = 3$, $|A \cap C| = 3$, $|B \cap C| = 4$, and $|A \cap B \cap C| = 2$. Thus $|A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = 5+5+8-3-3-4+2$ or 10, and Theorem 3 is verified. ♦

(B) Prove

5 +10+ 15+ .... +5n=5n(n+1)/2

for all n ≥ 1; using Mathematical Induction

Sol==

for all $n \geq 1$,

using mathematical inductn.

soln: ⇒ for $n = 5$.

$L.H.S. = P(1) = 5 = 5.$

$R.H.S. = P(1) = \left(\frac{5 \times 1 (1+1)}{2}\right) = \left(\frac{5(2)}{2}\right)$

$= \frac{10}{2} = 5$

∴ $P(1)$ is true for $n = 5$

L.H. = CPK

Induction step: For $n = K$

$L.H.S. = P(K) = 5 \times 1 + 5 \times 2 + 5 \times 3 + 5 \times 4 + \dots + K \cdot n$

$= \left[\frac{K \times n (k+1)}{2}\right]$

$P(K) : RHS = \left[\frac{K \cdot n (n+1)}{2}\right]$

∴ $P(K)$ is true for $n = K$ ....(ii).

for $n = k+1$

$p(k+1) = LHS = 5 + 10 + 15 + \cdots + k + (k+1)$

$$= \left[\frac{k(k+1)}{2}\right] + (k+1).$$

$$= \frac{k^2(k+1) + 4(k+1)(k+1)}{4}$$

$$= \frac{(k+1)[k + 4(k+1)]}{4}$$

$$= \frac{(k+1)[k^2 + 4(k+1)]}{4}$$

$$= \frac{(k+1)(k+2)}{4}$$

$$= \left[\frac{(k+1)(k+2)}{2}\right]$$

$= R.H.S.$

(c) Prove that :
If A, B and C are Boolean matrices of compatible sizes then,
(A B) C = A (B C)

Sol== **Solution:**

Let us assume that

$A = [a_{ij}]\ m \times n$

$B = [\ b_{jk}]\ m \times n$

$C = [\ c_{kl}]\ m \times n$

$(A \vee B) \vee C = A \vee (B \vee C)$

i,e $(A+B)+C = A+(B+C)$

Now,

$A + B = [a_{ij}]\ m \times n + [b_{jk}]\ m \times n$

$= [a_{ij}] + b_{jk}]\ m \times n$

$(A + B) + C = [a_{ij} + b_{jk}\ ]\ m \times n + [\ c_{kl}]\ m \times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m \times n \longrightarrow (1)$

$(B + C) = [b_{jk}]\ m \times n + [c_{kl}]\ m \times n$

$=[b_{jk} + c_{kl}]\ m \times n$

$(B + C) + A = [a_{ij}]\ m \times n + [b_{jk} + c_{kl}]\ m \times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m \times n \longrightarrow \qquad (2)$

From equation (1) & (2)

We get $(A \vee B) \vee C = A \vee (B \vee C)$

i.e.

$$\boxed{(A \Theta B)\ \Theta C = A \Theta (B\ \Theta C)}$$

(d) Obtain conjunctive normal form of :
ù(P $\vee$ Q) $\Leftrightarrow$ (P $\wedge$ Q).

Proof:-

$$
\begin{aligned}
& 7(P \vee Q) \leftrightarrow (P \wedge Q) \\
& by\ R \leftrightarrow S \Leftrightarrow (R \to S) \wedge (S \to R) \\
& \Leftrightarrow [7(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to 7(P \vee Q)] \\
& \Leftrightarrow [77(P \vee Q) \vee (P \wedge Q)] \wedge [7(P \wedge Q) \vee 7(P \vee Q)] \quad \{P \to Q \Rightarrow 7P \vee Q \\
& \Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(7P \vee 7Q) \vee 7(P \vee Q)] \quad \{By\ Demorgans\ property\ \&\ 77P \Rightarrow P \\
& \Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(7P \vee 7Q \vee 7P) \wedge (7P \vee 7Q \vee 7Q)] \{By\ Distributive\ property \\
& \Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((7P \vee 7P) \vee 7Q) \wedge ((7Q \vee 7Q) \vee 7P) \{By\ Associative\ property \\
& \Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (7P \vee 7Q) \wedge (7Q \vee 7P) \quad \{P \vee P = P \\
& \Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (7P \vee 7Q) \wedge (7P \vee 7Q) \quad \{By\ commulative\ property \\
& \Leftrightarrow (P \vee Q) \wedge (7P \vee 7Q) \quad \{P \wedge P = P
\end{aligned}
$$

It is the form of product of elementary sum of min terms.

Hence it is form of Principal Conjunction Normal Form.

(c) Obtain Principal Disjunctive Normal Form of
P $\to$ ((P $\to$ Q), $\wedge$ **ù** (**ù**Q $\vee$ **ù** P)).

Sol== $\Rightarrow (7P \wedge T) \vee (Q \wedge T)$ **{by** P $\wedge$ T=P

$\Rightarrow [7P \wedge (Q \vee 7Q)] \vee [Q \wedge (P \vee 7P)]$ {by P $\vee$ 7P=T

$\Rightarrow [(7P \wedge Q) \vee (7P \wedge 7Q)] \vee [(Q \wedge P) \vee (Q \wedge 7P)]$ {by distributive property

$\Rightarrow (7P \wedge Q) \vee (7P \wedge 7Q) \vee (Q \wedge P) \vee (Q \wedge 7P)$ {by Associative property

$\Rightarrow (7P \wedge Q) \vee (7P \wedge 7Q) \vee (P \wedge Q) \vee (P \wedge 7Q)$ {by Commutative property

$\therefore$ It is form of sum of elementary product of min term.

Hence, it is in the form of Principal Disjunction Normal Form.

$$7(P \vee Q) \leftrightarrow (P \wedge Q)$$
$$by\, R \leftrightarrow S \Leftrightarrow (R \to S) \wedge (S \to R)$$
$$\Leftrightarrow [7(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to 7(P \vee Q)]$$
$$\Leftrightarrow [77(P \vee Q) \vee (P \wedge Q)] \wedge [7(P \wedge Q) \vee 7(P \vee Q)] \quad \{P \to Q \Rightarrow 7P \vee Q$$
$$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(7P \vee 7Q) \vee 7(P \vee Q)] \quad \{By\ Demorgans\ property\ \&\ 77P \Rightarrow P$$
$$\Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(7P \vee 7Q \vee 7P) \wedge (7P \vee 7Q \vee 7Q)] \{By\ Distributive\ property$$
$$\Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((7P \vee 7P) \vee 7Q) \wedge ((7Q \vee 7Q) \vee 7P) \{By\ Associative\ property$$
$$\Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (7P \vee 7Q) \wedge (7Q \vee 7P) \quad \{P \vee P = P$$
$$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (7P \vee 7Q) \wedge (7P \vee 7Q) \quad \{By\ commulative\ property$$
$$\Leftrightarrow (P \vee Q) \wedge (7P \vee 7Q) \quad \{P \wedge P = P$$

(b) Find an explicit formula for the sequence defined by $C_n = 6C_{n-1} + 7C_{n-2}$ with initial conditions
$C_0 = 2$, $C_1 = 1$.

Sol== First find sequence for recurrence relation

$a_n = 4a_{n-1} + 5a_{n-2}$

For n = 3 $a_3 = 4a_{3-1} + 5a_{3-2}$

$= 4a_2 + 5a_1$

$= 4(6) + 5(2)$

$= 24 + 10$

$= 34$

For n = 4 $a_4 = 4a_{4-1} + 5a_{4-2}$

$= 4a_3 + 5a_2$

$= 4(34) + 5(6)$

$= 166$

For n=5 $a_5 = 4a_{5-1} + 5a_{5-2}$

= $4a_4$ + $5a_3$

= 4(166) + 5(34)

=834

∴ Sequence is 2,6,34,166,834----

The recurrence relation a_n= $4a_{n-1}$ + $5a_{n-2}$ is linear homogeneous

Equation of degree 2.

It associated equation is

x^2 = 4x+5

Rewriting this as

x^2 -4x-5=0

x^2 -5x+x-5=0

(x-5)(x+1)=0

X=5 or x=-1

The roots of the equation is s_1 =5 ands_2 =-1

Now, by teorem(i)

We can find value of u and v

From $a_n = us_1^n + vs_2^n$ -----------(A)

For n=1

$a_1 = us_1 + vs_{12}$

2 = u(5) + v(-1)

2 = 5u – v ------------(i)

For n = 2

$a_2 = us_1^n + vs_2^n$

6 = u$(5)^2$ + v $(-1)^2$

6= 25u+v -----------(ii)

Solving equation (i) and (ii)

5u – v = 2

25u + v = 6

\+   +   +

30u   = 8

$\boxed{U=8/30}$

Putting values of u in equation (i)

2 = 5 (8/30) – v

2 = (8/6) – v

2 = 8/6 – 2

2 = 8-12/6

V = -4/6

$\boxed{V = -2/3}$

Put value of u_1, v_1 , s_2 and s_2 in equation (A)

$a_n = us_1^n + vs_2^n$

a_n = (8/30) $(5)^n$ + (-2/3) $(-1)^n$

a_n = 8/30 $(5)^n$ + -2/3 $(-1)^n$

∴Which is required formula.

(a) Define :
(i) Semigroup
(ii) Monoid
(iii) Subsemigroup
(iv) Group Homomorphism.

(i) Semigroup:-

Let S be a non-empty set and * be a binary operation on S. The algebraic system (S, *) is called a semigroup if the operation * is

(1) The operation * is a closed operation on set A.

(2) The operation * is an associative operation.

Or (S, *) is a semigroup if for any x, y, z ϵ S,

$$\boxed{(x * y) * z = x * (y * z)}$$

Free semigroup:

If * is an associative binary operation, and (A,*) is a semigroup. The semigroup(A,*) is called free semigroup by A.

Ex:

Consider an algebraic system (S,*) where S = {1,2,3,5,7,9----} the set of all positive odd integers and * is a binary operation means multiplication. Determine whether (S,*) is a semigroup.

(ii) Monoid:-

Let us consider an algebraic system (M, *), where * is a binary operation on M. Then the system (M, *) is said to be a monoid if it satisfies the following properties:

(1) The operation * is a closure operation on set A.
(2) The operation * is an associative operation.
(3) There exists an identify element w. r. t. The operation *.

Ex:-

Consider an algebra system (N, +), where the set N = {0,1,2,3----} the set of natural numbers and + is an addition operation. Determine whether (N,+) is a monoid.

(iii) Subsemigroup:-

Let (S,*) be a semigroup and T $\subseteq S$, if the set T is closed number the operation * then (T,*) is said to be subsemigroups of (S,*).

Ex:

Consider a semigroup (N,+), where N is the sset of all natural number and + is an addition operation.

The algebric system (E,+) is a subsemigroup of (N,+), where E is a set of all +ve even integer.

(iv) Group homomorphism:-

Let (S,*) and (T,*) be two semigroups. An everywhere defined function f: $S \to T$ is called a homomorphism from (S,*) and (T,*)

If (a * b) = f(a) *f (b)

For all a and b in S.

If f is also onto.

We say that T is a homomorphic image of S.

(b)Let the number of edges of graph G be m. Then G has a Hamiltonian circuit if
m ≥ 1/2 (n2 – 3n + 6) where n = n0 of vertices.

Sol== partial order set :

Let A is a relation or set A .then relation R is called partial order. If it is reflexive, antisymentric and transitive.

If R is a partial order relation on set A. then set A together with partial order relation R is know as partial orderd set or partial order set.

Ex. Let Z be a set f integers "$\leq$" be a relation on Z.

$\therefore$ Reflexive property is satisfied .

($\because$ $a \leq a$ $\forall$ $a \in z$)

Let $a,b \in z$

$a \leq b$ and $b \leq a \Rightarrow a=b$

$\therefore$ Antisymmentric property is satisfied

$a \leq b$ and $b \leq c \Rightarrow a \leq c$

$\therefore$ Transitive property is satiesfide.

$\therefore$"$\leq$" is a partial oreder relation on Z

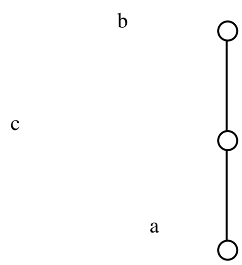
Similarly "$\geq$"is also a partial oreder relation on Z.

Chain order set:

If every pair of element in a poset is comparable than poset A is called linear order set .Or set A is chain .

Ex. A={a,b,c}

$a \leq c$, $c \leq b$



This order is in linear or chain.

Hence it is called chain or linear orderd.

Lexicographic:

Let A×B is a cartesion product of two sets A &B .we define "<"as follow.

(a b) < (a' b') if a< a' or if a=a' then b< b'

This is used in dictionary.

Hence it is also as dictionary

Ex. Help, help

Help< help

Isomorphism:

Let ($A \leq$) and ($A' \leq'$) be posets and let f: A $\rightarrow$ A!be a one-to-one correspondence between f: A & A! The function f is called an Isomorphism from A to A'

It for any a, b $\in$ A, $a \leq b$

$\Leftrightarrow$ $f(a) \leq' a(b)$.

(Proof left)

(c) Let G be the set of all non-zero real numbers and let
ab
a * b = . Show that (G, *) is an Abelian
group.

Sol== **To show:** (G,*) is an abelian group.

Closure property:

The set G is closed under the operation *.

Since, a*b = $\frac{ab}{2}$ is a real number.

Hence, belongs to G.

Associative property:

The operation * is associative.

Let a, b, c ∈ G, then

We have

$$(a*b)*c = \left(\frac{ab}{2}\right)*c$$

$$= \frac{(ab)c}{4}$$

$$= \frac{abc}{4}$$

Similarly, a* (b*c) * a = $\left(\frac{ab}{2}\right)$

$$= \frac{a(bc)}{4}$$

$$= \frac{abc}{4}$$

(d) Let T be the set of all even integers. Show that the semigroup (z, +) and (T, +) are isomorphic

Sol== solu:

Let a and d be any element in G, since R is an equivalence relation b ∈ [a]

If and only if [b] = [a]

Also G/R is a group

Therefore [b] =[a] if and only if

[e] = [a^{-1}] [b]

= [a^{-1} b]

Thus, b ∈ [a] if and only if

H = [e] =[a^{-1} b]

That is, b ∈ [a] if and only if

a^{-1}b∈ H or b ∈ aH

This prove that

[a] = aH for every a ∈G

We can show

Similar that b ∈ [a] if and only if

H = [e]

= [b] $[a]^{-1}$

= $[ba]^{-1}$

This is equivalent to the statement [a] = Ha

Thus, [a] = are isomorphic

Either

1.(A) (a) Let A, B and C be finite sets with | A | = 6, | B | = 8, | C | = 6, | A ∗ B ∗ C | = 11, | A) B | = 3, | A) C | = 2 and | B) C | = 5. Find | A) B) C |.

Basic of induction: For n=1

P (1) = LHS = A1$\cap B$

P (1) = LHS = A1$\cap B$

$\therefore LHS =$ RHS

A1∩B = A1∩B

P (1) is true for n=1

Induction step: For n=k

P (k)= LHS = $\left(\bigcup_{i=1}^{k} Ai\right) \cap B = \bigcup_{i=1}^{k}(Ai \cap B)$

P (k) = RHS = $\bigcup_{i=1}^{k}(Ai \cap B)$

$\therefore$ p(k) is also true for n=k

Similarly for n = k+1

LHS = $\left(\bigcup_{i=1}^{k+1} Ai\right) \cap B$

= (A1 ∪ A2 ∪--------- ∪ Ak ∪ Ak +1) $\cap B$

= ($\left(\bigcup_{i=1}^{k} Ai\right)$ ∪ Ak +1) $\cap B$

= ($\left(\bigcup_{i=1}^{k} Ai\right) \cap B$) ∪ (Ak+ 1 $\cap B$) {by distributive property}

= ($\bigcup_{i=1}^{k}(Ai \cap B)$) ∪ (Ak+1 $\cap B$)

(c) Prove by mathematical induction :
3
$1^2 + 3^2 + 5^2 + ... + (2n – 1)^2 = n(2n +1) (2n – 1)$.

Sol==$a_n= 4a_{n-1} + 5a_{n-2}$ where $a_1=2$,$a_2 = 6$

Soln: First find sequence for recurrence relation

$a_n= 4a_{n-1} + 5a_{n-2}$

For n = 3 $a_3= 4a_{3-1} + 5a_{3-2}$

$= 4a_2 + 5a_1$

$= 4(6) + 5(2)$

$= 24 + 10$

$= 34$

For n = 4 $a_4= 4a_{4-1} + 5a_{4-2}$

$= 4a_3 + 5a_2$

$= 4(34) + 5(6)$

$= 166$

For n=5 $a_5= 4a_{5-1}+5a_{5-2}$

$= 4a_4 + 5a_3$

$= 4(166) + 5(34)$

$=834$

∴ Sequence is 2,6,34,166,834----

The recurrence relation $a_n= 4a_{n-1} + 5a_{n-2}$ is linear homogeneous

Equation of degree 2.

It associated equation is

$x^2 = 4x+5$

Rewriting this as

$x^2 -4x-5=0$

$x^2 -5x+x-5=0$

(x-5)(x+1)=0

X=5 or x=-1

The roots of the equation is s_1 =5 ands_2 =-1

Now, by theorem(i)

We can find value of u and v

From $a_n = us_1^n + vs_2^n$----------- (A)

For n=1

$a_1 = us_1 + vs_{12}$

2 = u(5) + v(-1)

2 = 5u – v ------------(i)

For n = 2

$a_2 = us_1^n + vs_2^n$

6 = u$(5)^2$ + v $(-1)^2$

6= 25u+v -----------(ii)

Hence proved

(c) Prove that :
If A, B and C are Boolean matrices of compatible sizes then,
(A B) C = A (B C)

Sol== **Solution:**

Let us assume that

A = [a_{ij}] m × n

B = [b_{jk}] m× n

C= [c_{kl}] m× n

(A ∨ B) ∨ C =A ∨ (B ∨ C)

i,e (A+B)+C = A+(B+C)

Now,

A + B = [a_{ij}] m X n + [b_{jk}] m× n

= [a_{ij}] + b_{jk}] m× n

$(A + B) + C = [a_{ij} + b_{jk}]\ m \times n + [c_{kl}]\ m \times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m \times n \longrightarrow$ (1)

$(B + C) = [b_{jk}]\ m \times n + [c_{kl}]\ m \times n$

$= [b_{jk} + c_{kl}]\ m \times n$

$(B + C) + A = [a_{ij}]\ m \times n + [b_{jk} + c_{kl}]\ m \times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m \times n \longrightarrow$ (2)

From equation (1) & (2)

We get $(A \vee B) \vee C = A \vee (B \vee C)$

i.e.

$$\boxed{(A \Theta B)\ \Theta C = A \Theta (B\ \Theta C)}$$

EITHER

2. (a) Obtain principle disjunctive normal form of :

(i) (P (Q) ϶ (Q (R) ϶ (P (R)

(ii) P □ Q □ R

$\Rightarrow [(P \vee Q)\neg[\neg P \wedge (\neg Q \vee \neg R)]] \vee \neg[(\neg P \wedge \neg Q) \vee \neg(P \wedge \neg R)]$ {by associative property

$\Rightarrow [(P \vee Q) \wedge \neg(\neg P \wedge \neg(Q \vee R))] \vee [(\neg P \wedge \neg Q \vee \neg R)]$ {By demorgans and distributive prop. Respt.

$\Rightarrow [(P \vee Q) \wedge \neg\neg(P \vee (Q \wedge R))] \vee (\neg P \wedge \neg (Q \wedge R))$ {bydemorgans property

$\Rightarrow [(P \vee Q) \wedge (P \vee (Q \wedge R)] \vee \neg (P \vee (Q \wedge R)$ { by demorgans property & $\neg\neg A \Rightarrow A$

$\Rightarrow [P \vee (Q \wedge (Q \wedge R))] \vee \neg(P \vee (Q \wedge R)]$ {by distributive property.

$\Rightarrow [P \vee (Q \wedge Q) \wedge R) \vee \neg(P \vee Q \wedge R)]$ {by associative property

$\Rightarrow (P \vee (Q \wedge R)) \vee \neg(P \vee (Q \wedge R))$ {by idempotent property & $P \wedge P \Rightarrow P$

$\Rightarrow T$ {$P \vee \neg P \Rightarrow T$

Hence Proved

A. Obtain the Principal Disjunction Normal Form of :

1. $\neg P \vee Q$

2. $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

<u>Solution</u>:

1. $\neg P \vee Q$

$\Rightarrow (\neg P \wedge T) \vee (Q \wedge T)$ {**by** $P \wedge T = P$

$\Rightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$ {by $P \vee \neg P = T$

$\Rightarrow [(\neg P \wedge Q) \vee (\neg P \wedge \neg Q)] \vee [(Q \wedge P) \vee (Q \wedge \neg P)]$ {by distributive property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$ {by Associative property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (P \wedge \neg Q)$ {by Commutative property

$\therefore$ It is form of sum of elementary product of min term.

Hence,it is in the form of Principal Disjunction Normal Form.

(d) Obtain conjunctive normal form of :

$\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$.

Proof:-

$\neg(P \vee Q) \leftrightarrow (P \wedge Q)$

$by\, R \leftrightarrow S \Leftrightarrow (R \to S) \wedge (S \to R)$

$\Leftrightarrow [\neg(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to \neg(P \vee Q)]$

$\Leftrightarrow [\neg\neg(P \vee Q) \vee (P \wedge Q)] \wedge [\neg(P \wedge Q) \vee \neg(P \vee Q)] \quad \{P \to Q \Rightarrow \neg P \vee Q$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(\neg P \vee \neg Q) \vee \neg(P \vee Q)] \quad \{By\ Demorgans\ property\ \&\ \neg\neg P \Rightarrow P$

$\Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q)]\{By\ Distributive\ property$

$\Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((\neg P \vee \neg P) \vee \neg Q) \wedge ((\neg Q \vee \neg Q) \vee \neg P)\{By\ Associative\ property$

$\Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (\neg P \vee \neg Q) \wedge (\neg Q \vee \neg P) \quad \{P \vee P = P$

$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q) \quad \{By\ commulative\ property$

$\Leftrightarrow (P \vee Q) \wedge (\neg P \vee \neg Q) \quad \{P \wedge P = P$

It is the form of product of elementary sum of min terms.

Hence it is form of Principal Conjunction Normal Form.

(c) Obtain Principal Disjunctive Normal Form of
P → ((P → Q), ∧ ù (ùQ ∨ ù P)).

Sol== $\Rightarrow (\neg P \wedge T) \vee (Q \wedge T)$ **{by** P $\wedge$ T=P

$\Rightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$ {by P $\vee$ $\neg$P=T

$\Rightarrow [(\neg P \wedge Q) \vee (\neg P \wedge \neg Q)] \vee [(Q \wedge P) \vee (Q \wedge \neg P)]$ {by distributive property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$ {by Associative property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (P \wedge \neg Q)$ {by Commutative property

$\therefore$ It is form of sum of elementary product of min term.

Hence, it is in the form of Principal Disjunction Normal Form.

$\neg(P \vee Q) \leftrightarrow (P \wedge Q)$

$by\, R \leftrightarrow S \Leftrightarrow (R \to S) \wedge (S \to R)$

$\Leftrightarrow [\neg(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to \neg(P \vee Q)]$

$\Leftrightarrow [\neg\neg(P \vee Q) \vee (P \wedge Q)] \wedge [\neg(P \wedge Q) \vee \neg(P \vee Q)] \quad \{P \to Q \Rightarrow \neg P \vee Q$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(\neg P \vee \neg Q) \vee \neg(P \vee Q)] \quad \{By\ Demorgans\ property\ \&\ \neg\neg P \Rightarrow P$

$\Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q)]\{By\ Distributive\ property$

$\Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((\neg P \vee \neg P) \vee \neg Q) \wedge ((\neg Q \vee \neg Q) \vee \neg P)\{By\ Associative\ property$

$\Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (\neg P \vee \neg Q) \wedge (\neg Q \vee \neg P) \quad \{P \vee P = P$

$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q) \quad \{By\ commulative\ property$

$\Leftrightarrow (P \vee Q) \wedge (\neg P \vee \neg Q) \quad \{P \wedge P = P$

(b) Find an explicit formula for the sequence defined by $C_n = 6C_{n-1} + 7C_{n-2}$ with initial conditions
$C_0 = 2$, $C_1 = 1$.

Sol== First find sequence for recurrence relation

$a_n = 4a_{n-1} + 5a_{n-2}$

For n = 3 $a_3 = 4a_{3-1} + 5a_{3-2}$

$= 4a_2 + 5a_1$

= 4(6) + 5(2)

= 24 + 10

= 34

For n = 4 $a_4 = 4a_{4-1} + 5a_{4-2}$

$= 4a_3 + 5a_2$

= 4(34) + 5(6)

= 166

For n=5 $a_5 = 4a_{5-1} + 5a_{5-2}$

$= 4a_4 + 5a_3$

= 4(166) + 5(34)

=834

∴ Sequence is 2,6,34,166,834----

The recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ is linear homogeneous

Equation of degree 2.

It associated equation is

$x^2 = 4x+5$

Rewriting this as

$x^2 - 4x - 5 = 0$

$x^2 - 5x + x - 5 = 0$

(x-5)(x+1)=0

X=5 or x=-1

The roots of the equation is s_1 =5 ands_2 =-1

Now, by teorem(i)

We can find value of u and v

From $a_n = us_1^n + vs_2^n$ -----------(A)

For n=1

$a_1 = us_1 + vs_{12}$

2 = u(5) + v(-1)

2 = 5u – v ------------(i)

For n = 2

$a_2 = us_1^n + vs_2^n$

6 = u$(5)^2$ + v $(-1)^2$

6= 25u+v -----------(ii)

Solving equation (i) and (ii)

5u – v = 2

25u + v = 6

\+ + +

30u = 8

$\boxed{\text{U=8/30}}$

Putting values of u in equation (i)

2 = 5 (8/30) – v

2 = (8/6) – v

2 = 8/6 – 2

2 = 8-12/6

V = -4/6

$\boxed{\text{V = -2/3}}$

Put value of u_1, v_1 , s_2 and s_2 in equation (A)

$a_n = us_1^n + vs_2^n$

a_n = (8/30) $(5)^n$ + (-2/3) $(-1)^n$

a_n = 8/30 $(5)^n$ + -2/3 $(-1)^n$

∴Which is required formula.

(a) Define :
(i) Semigroup
(ii) Monoid
(iii) Subsemigroup
(iv) Group Homomorphism.

(i) Semigroup:-

Let S be a non-empty set and * be a binary operation on S. The algebraic system (S, *) is called a semigroup if the operation * is

(1) The operation * is a closed operation on set A.

(2) The operation * is an associative operation.

Or (S, *) is a semigroup if for any x, y, z ϵ S,

$$\boxed{(x * y) * z = x * (y * z)}$$

Free semigroup:

If * is an associative binary operation, and (A,*) is a semigroup. The semigroup(A,*) is called free semigroup by A.

Ex:

Consider an algebraic system (S,*) where S = {1,2,3,5,7,9----} the set of all positive odd integers and * is a binary operation means multiplication. Determine whether (S,*) is a semigroup.

(ii) Monoid:-

Let us consider an algebraic system (M, *), where * is a binary operation on M. Then the system (M, *) is said to be a monoid if it satisfies the following properties:

(1) The operation * is a closure operation on set A.
(2) The operation * is an associative operation.
(3) There exists an identify element w. r. t. The operation *.

Ex:-

Consider an algebra system (N, +), where the set N = {0,1,2,3----} the set of natural numbers and + is an addition operation. Determine whether (N,+) is a monoid.

(iii) Subsemigroup:-

Let (S,*) be a semigroup and T $\subseteq S$, if the set T is closed number the operation * then (T,*) is said to be subsemigroups of (S,*).

Ex:

Consider a semigroup (N,+), where N is the sset of all natural number and + is an addition operation.

The algebric system (E,+) is a subsemigroup of (N,+), where E is a set of all +ve even integer.

(iv) Group homomorphism:-

Let (S,*) and (T,*) be two semigroups. An everywhere defined function f: $S \rightarrow T$ is called a homomorphism from (S,*) and (T,*)

If (a * b) = f(a) *f (b)

For all a and b in S.

If f is also onto.

We say that T is a homomorphic image of S.

(b)Let the number of edges of graph G be m. Then G has a Hamiltonian circuit if
m ≥ 1/2 (n2 – 3n + 6) where n = n0 of vertices.

Sol== <u>partial order set :</u>

Let A is a relation or set A .then relation R is called partial order. If it is reflexive, antisymmentric and transitive.

If R is a partial order relation on set A. then set A together with partial order relation R is know as partial orderd set or partial order set.

Ex. Let Z be a set f integers "≤" be a relation on Z.

$\because$ Reflexive property is satisfied .

($\because$ a≤a $\forall$ a$\in$ z)

Let a,b$\in$ z

a≤ b and b ≤ a $\Rightarrow$ a=b

$\therefore$ Antisymmentric property is satisfied

a≤ b and b ≤ c $\Rightarrow$ a≤ c

$\therefore$ Transitive property is satiesfide.

$\therefore$"≤" is a partial oreder relation on Z

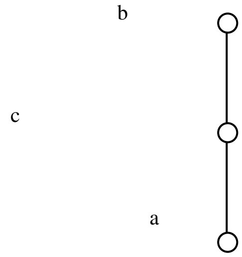
Similarly "≥"is also a partial oreder relation on Z.

<u>Chain order set:</u>

If every pair of element in a poset is comparable than poset A is called linear order set .Or set A is chain .

Ex. A={a,b,c}

$a \le c,\ c \le b$



This order is in linear or chain.

Hence it is called chain or linear orderd.

<u>Lexicographic:</u>

Let A×B is a cartesion product of two sets A &B .we define “<”as follow.

(a b) < (a’ b’) if a< a’ or if a=a’ then b< b’

This is used in dictionary.

Hence it is also as dictionary

Ex. Help, help

Help< help

<u>Isomorphism:</u>

Let (A≤) and (A’≤’) be posets and let f: A $\rightarrow$ A!be a one-to-one correspondence between f: A & A! The function f is called an Isomorphism from A to A’

It for any a, b $\in$ A, a≤ b

$\Leftrightarrow$ f(a) ≤’ a(b).

(Proof left)

(c) Let G be the set of all non-zero real numbers and let
ab
a * b = . Show that (G, *) is an Abelian
group.

Sol== **To show:** (G,*) is an abelian group.

Closure property:

The set G is closed under the operation *.

Since, a*b = $\frac{ab}{2}$ is a real number.

Hence, belongs to G.

Associative property:

The operation * is associative.

Let a, b, c ∈ G, then

We have

$$(a*b)*c = \left(\frac{ab}{2}\right)*c$$

$$= \frac{(ab)c}{4}$$

$$= \frac{abc}{4}$$

Similarly, a* (b*c) * a = $\left(\frac{ab}{2}\right)$

$$= \frac{a(bc)}{4}$$

$$= \frac{abc}{4}$$

(d) Let T be the set of all even integers. Show that the semigroup (z, +) and (T, +) are isomorphic

Sol== solu:

Let a and d be any element in G, since R is an equivalence relation b ∈ [a]

If and only if [b] = [a]

Also G/R is a group

Therefore [b] =[a] if and only if

[e] = [a^{-1}] [b]

= [a^{-1} b]

Thus, b ∈ [a] if and only if

H = [e] =[a^{-1} b]

That is, b ∈ [a] if and only if

a^{-1}b∈ H or b ∈ aH

This prove that

[a] = aH for every a ∈G

We can show

Similar that b ∈ [a] if and only if

H = [e]

= [b] $[a]^{-1}$

= $[ba]^{-1}$

This is equivalent to the statement [a] = Ha

Thus, [a] = are isomorphic

1] (A) Let $U = \{a, b, c, d, e, f, g, h, k\}, A = \{a, b, c, g\}, B = \{d, e, f, g\}, C = \{a, c, f\} and\ D$= {f,h,k}.

Compute:

(i) $A \oplus B$ **(ii)** C$\oplus D$

(iii) $\overline{A \cup B}$ **(iv)** $\overline{C \cap D}$

Solution:

(B) (i) Construct truth table for statement:

p$\Rightarrow q \Leftrightarrow \neg p \vee q$

soln :

(1) (2) (3) (4) (5)

P	Q	(P$\Rightarrow Q$)	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From (3) and (5) column

$\therefore$p$\Rightarrow q \Leftrightarrow \neg p \vee q$ Hence proved.

(ii) Prove that if m^2 is odd then m is odd.
Soln:

OR

(c) Explain principle of mathematical induction and use induction method to

Prove:

$$\left(\bigcup_{i=1}^{n} Ai\right) \cap B = \bigcup_{i=1}^{n} (Ai \cap B)$$

Soln:

We can prove by mathematical induction.

Basic of induction: For n=1

P(1) = LHS = A1$\cap B$

P(1) = LHS = A1$\cap B$

$\therefore LHS =$ RHS

A1∩B = A1∩B

P(1) is true for n=1

Induction step: For n=k

P(k)= LHS = $\left(\bigcup_{i=1}^{k} Ai\right) \cap B = \bigcup_{i=1}^{k} (Ai \cap B)$

P(k) = RHS = $\bigcup_{i=1}^{k}(Ai \cap B)$

∴p(k) is also true for n=k

Similarly for n = k+1

LHS = $\left(\bigcup_{i=1}^{k+1} Ai\right) \cap B$

= (A1 ∪ A2 ∪--------- ∪Ak ∪Ak+1) ∩ B

= $\left(\left(\bigcup_{i=1}^{k} Ai\right) \cup Ak+1\right) \cap B$

=$\left(\left(\bigcup_{i=1}^{k} Ai\right) \cap B\right) \cup (Ak+1 \cap B)$ {by distributive property}

= $\left(\bigcup_{i=1}^{k}(Ai \cap B)\right) \cup (Ak+1 \cap B)$

= $\bigcup_{i=1}^{k+1}(Ai \cap B)$

= RHS

Thus the implication p(k)→p(k+1) is a tautology

∴ By principle of Mathematic Induction

P(n) is true for all n≥1

Hence proved

$$\boxed{\left(\bigcup^{n} Ai\right) \cap B = \bigcup_{i=1}^{n}(Ai \cap B)}$$

(D) Solve the recurrence relation:

$a_n = 4a_{n-1} + 5a_{n-2}$ where a_1=2 ,a_2 = 6

Soln: First find sequence for recurrence relation

$a_n = 4a_{n-1} + 5a_{n-2}$

For n = 3 $a_3 = 4a_{3-1} + 5a_{3-2}$

= $4a_2 + 5a_1$

= 4(6) + 5(2)

= 24 + 10

= 34

For n = 4 $a_4 = 4a_{4-1} + 5a_{4-2}$

= $4a_3 + 5a_2$

= 4(34) + 5(6)

= 166

For n=5 $a_5 = 4a_{5-1} + 5a_{5-2}$

= $4a_4 + 5a_3$

= 4(166) + 5(34)

=834

∴ Sequence is 2,6,34,166,834----

The recurrence relation a_n= $4a_{n-1}$ + $5a_{n-2}$ is linear homogeneous

Equation of degree 2.

It associated equation is

x^2 = 4x+5

Rewriting this as

x^2 -4x-5=0

x^2 -5x+x-5=0

(x-5)(x+1)=0

X=5 or x=-1

The roots of the equation is s_1 =5 ands_2 =-1

Now, by theorem(i)

We can find value of u and v

From a_n = us_1^n + vs_2^n----------- (A)

For n=1

a_1 = us_1 + vs_{12}

2 = u(5) + v(-1)

2 = 5u – v ------------(i)

For n = 2

a_2 = us_1^n + vs_2^n

6 = u$(5)^2$ + v $(-1)^2$

6= 25u+v -----------(ii)

Solving equation (i) and (ii)

5u – v =2

25u + v =6

\+ \+ \+

30u = 8

$\boxed{U=8/30}$

Putting values of u in equation (i)

2 = 5 (8/30) – v

2 = (8/6) – v

2 = 8/6 – 2

2 = 8-12/6

V = -4/6

$\boxed{V = -2/3}$

Luke of

Put value of u_1, v_1 , s_2 and s_2 in equation (A)

$a_n = us_1^n + vs_2^n$

a_n = (8/30) $(5)^n$ + (-2/3) $(-1)^n$

a_n = 8/30 $(5)^n$ + -2/3 $(-1)^n$

∴Which is required formula?

Q.2

Either

A. Explain dual formula and show that if A(P,Q,R)= $\neg$P $\wedge$ $\neg$(Q $\vee$ R) then

i. A($\neg$P,$\neg$Q,$\neg$R) $\Leftrightarrow$ $\neg$A *(P,Q,R)

ii. $\neg$A(P,Q,R) $\leftrightarrow$ A*($\neg$P,$\neg$Q,$\neg$R)

<u>Solution:</u>

(i) A($\neg$P,$\neg$Q,$\neg$R) $\Leftrightarrow$ $\neg$A *(P,Q,R)

We have to prove that, if A(P,Q,R)=$\neg$P $\wedge$ $\neg$(Q $\vee$ R) then

A($\neg$P,$\neg$Q,$\neg$R) $\Leftrightarrow$ $\neg$A *(P,Q,R)

$\Rightarrow$ A(P,Q,R) = $\neg$P $\wedge$ $\neg$(Q $\vee$ R)

= ($\neg$P $\wedge$ $\neg$Q $\wedge$ $\neg$R)

= $\neg$ (P $\vee$ Q $\vee$ R)

$\Rightarrow$ A($\neg$P,$\neg$Q,$\neg$R) = (P $\vee$ Q $\vee$ R) $\longrightarrow$ (1)

$\Rightarrow$ A*(P,Q,R)=$\neg$P $\wedge$ $\neg$(Q $\vee$ R)

= ($\neg$P $\wedge$ $\neg$Q $\wedge$ $\neg$R)

= $\neg$ (P $\vee$ Q $\vee$ R)

$\Rightarrow$ $\neg$A*(P,Q,R) = (P $\vee$ Q $\vee$ R) $\longrightarrow$ (2)

From eq 1 & 2

We get, $\boxed{\text{A}(\neg\text{P},\neg\text{Q},\neg\text{R}) \Leftrightarrow \neg\text{A}^*(\text{P},\text{Q},\text{R})}$

Hence proved

(ii) We have to prove that , if A(P,Q,R)= $\neg$P $\wedge$ $\neg$(Q $\vee$ R) then

$\neg$A(P,Q,R) $\leftrightarrow$ A*($\neg$P,$\neg$Q,$\neg$R)

$\Rightarrow A(P,Q,R) = \neg P \wedge \neg(Q \vee R)$

$= \neg(P \vee (Q \vee R))$

$\Rightarrow \neg A(P,Q,R) = \neg\neg(P \vee (Q \vee R))$

$= P \vee (Q \vee R)$

$\Rightarrow A^*(P,Q,R) = \neg P \vee \neg(Q \wedge R) \longrightarrow (1)$

$\Rightarrow A^*(\neg P,\neg Q,\neg R) = P \vee \neg(\neg Q \wedge \neg R)$

$= P \vee \neg(\neg(Q \wedge R))$

$\Rightarrow A^*(\neg P,\neg Q,\neg R) = P \vee (Q \vee R) \longrightarrow (2)$

From eq 1 & 2

We get,

$\neg A(P,Q,$ $\boxed{\neg A(P,Q,R) \leftrightarrow A^*(\neg P,\neg Q,\neg R)}$

Hence proved

A. Obtain the Principal Disjunction Normal Form of :

1. $\neg P \vee Q$

2. $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

<u>Solution</u>:

1. $\neg P \vee Q$

$\Rightarrow (\neg P \wedge T) \vee (Q \wedge T)$ **{by** $P \wedge T = P$

$\Rightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$ {by $P \vee \neg P = T$

$\Rightarrow [(\neg P \wedge Q) \vee (\neg P \wedge \neg Q)] \vee [(Q \wedge P) \vee (Q \wedge \neg P)]$ {by distributive property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$ {by Associative property

$\Rightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (P \wedge \neg Q)$ {by Commutative property

$\therefore$ It is form of sum of elementary product of min term.

Hence,it is in the form of Principal Disjunction Normal Form.

2. $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$\Rightarrow [(P \wedge Q) \wedge T] \vee [(\neg P \wedge R) \wedge T] \vee [(Q \wedge R) \wedge T]$

{By $P \vee \neg P = T$

$\Rightarrow [(P \wedge Q) \wedge (R \vee \neg R)] \vee [(\neg P \wedge R) \wedge (Q \vee \neg Q)] \vee [(Q \wedge R) \wedge (P \vee \neg P)]$ {by $P \vee \neg P = T$

$\Rightarrow [(P \wedge Q \wedge R)] \vee [(P \wedge Q \wedge R)] \vee [(\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q)] \vee [(Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P)]$ {

by distributive property

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)$

{by Commutative property

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$ {by $P \vee P = P$

$\therefore$ it is form of sum of elementary product of min term.

Hence,It is in the form of Principal Disjunction Normal Form.

OR

(C) Determine whether the conclusion C follows logically from premises H1and H2:

(i) H1 : $P \rightarrow Q$ H2 : $\neg P$ C : Q

(ii) H1: $P \rightarrow Q$ H2:$\neg(p \wedge Q)$C:$\neg$ P

(iii) H1:$\neg$ P H2: $P \rightleftharpoons Q$ C:$\neg(p \wedge Q)$

(iv) H1: $P \rightarrow Q$ H2: Q C:P

(D) Show that:

(X) (P(X) ∨ Q(X)) ⟹ (X) P(X) ∨ (∃X) Q(X)

Q.3)

(A) (i) If $A \subseteq C$ and $B \subseteq D$ then prove that

$A \times B \subseteq C \times D$

Soln :-

Given that If $A \subseteq C$ and $B \subseteq D$

To prove: $A \times B \subseteq C \times D$

Proof: Let $(x,y) \in A \times B$, then $x \in A$ and $y \in B$

Since $A \subseteq C$ and $B \subseteq D$, $x \in C$ and $x \in D$

Hence, $(x,y) \in C \times D$

Then $\boxed{A \times B \subseteq C \times D}$

Hence proved

(iii) Let A={1,2,4}, B={2,5,7} and c={1,3,7}

Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(B) (ii) Let R and S are relation from A to B then prove that:

(i) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

Ans: (i) Let $(a,b) \in (R \cap S)^{-1}$

So, we have $(b,a) \in (R \cap S)$

Now $(b,a) \in R$ and $(b,a) \in S$

This mean (a,b) ∈ R^{-1} and (a,b) ∈ S^{-1}

Hence (a,b) ∈ $R^{-1} \cap S^{-1}$ ---------(i)

Conversely,

Let, (a,b) ∈ $R^{-1} \cap S^{-1}$

So, we have (a,b) ∈ R^{-1} and (a,b) ∈ S^{-1}

This means (b,a) ∈ R and (b,a) ∈ S

So, (b,a) ∈ (R ∩ S) ---------(ii)

From (i) and (ii) we have

$(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

Hence Proved

(ii)$\overline{R \cap S} = \overline{R} \cup \overline{S}$

OR

(C)Let A = B ={1,2,3,4}, R={(1,1),(1,3),(2,3),(3,1),(4,2),(4,4)}

And S = {(1,2), (2,3),(3,1), (3,2), (3,1), (3,2), (4,3)} then compute

$\boldsymbol{M_{R\cap S}}$, $\boldsymbol{M_{R\cup S}}$, $^{\mathrm{M}}\boldsymbol{R^{-1}}$, , $^{\mathrm{M}}\boldsymbol{S^{-1}}$.

(D) Let A be a set with $|A|$ = n and let R be a relation on A. Then prove that:

$\boldsymbol{R^{\infty} = R \cup R^2 \cup \ldots\ldots \cup R^n}$

EITHER

4 (A) (i) Explain:

Post,chain, Hasses Diagram and draw theHasses diagram of posset A with

5 (A)Let (G *) and (G' *') be two group and let f : G→ G' be a homomorphism from G to G' then

(i) f(e)=e' where e is identity of G and e' is identity of G'

(ii) f(a^{-1}) =$(f(a))^{-1}$

solun: (a) Let x=f(e) then

x *' x = f(e) *' f(e)

= f(e * e)

= f(e)

= x

So, x*' x=x

Multiplying both side by x^{-1}

On right, we obtain

X = x*' x *' x^{-1} = x*' x^{-1} = e'

Thus f(e) = e'

(b) $a *' a^{-1} = e$

So, $f(a *' a^{-1}) = f(e)$

$= e'$ by part(a)

Or $f(a) *' f(a^{-1}) = e's$

Since, f is a homomorphism

Similarly, $f(a^{-1}) *' f(a) = e'$

Hence $f(a^{-1}) = (f((a))^{-1}$

(B) (i) Let G be an abeliangroup with identity e and let H={x:x^2=e}. Show thatH is a subgroup of G.

(ii)Insertion of two sub subgroup of G is a subgroup of G

OR

(C) Define:

Finite state Machine, state transition function and Moore Machine and

Draw diagraph whose table is: (summer-13)

(D) Let R be a congruence relation on a group G and let H={e}, the equivalent class containing the identity. Then H is a normal subgroup of G and for each

a∈G[a] = Ha = aH prove this

solu:

Let a and d be any element in G, since R is an equivalence relation b ∈ [a]

If and only if [b] = [a]

Also G/R is a group

Therefore [b] =[a] if and only if

[e] = [a^{-1}] [b]

= [a^{-1} b]

Thus, b ∈ [a] if and only if

H = [e] =[a^{-1} b]

That is, b ∈ [a] if and only if

a^{-1}b∈ H or b ∈ aH

This prove that

[a] = aH for every a ∈G

We can show

Similar that b ∈ [a] if and only if

H = [e]

= [b] $[a]^{-1}$

$= [ba]^{-1}$

This is equivalent to the statement $[a] = Ha$

Thus, $[a] = aH = Ha$ and H is normal.

1. EITHER

(A) Prove by mathematical induction that, for all

$$n! \geq 1,$$
$$n! \geq 2^{n-1}, where$$
$$1! = 1 and\ n! = n(n-1)!$$

Solution:-

(B) If a and b are two positive integers, then prove that

GCD(a,b).LCM(a,b) = ab

Verify above result for a = 100, b = 80

Proof:-

Let p1, p2, p3-----pn be the prime factor of a and b.

$$\left.\begin{array}{l} \therefore a = p1^{a1} * p2^{a2} * p3^{a3} * ----* pn^{an} \\ \therefore b = p1^{b1} * p2^{b2} * p3^{b3} * ----* pn^{bn} \end{array}\right\} \rightarrow (1)$$

By definition of LCM and GCD, we get

$$GCD(a,b) = p1^{\min(a1,b1)} * p2^{\min(a2,b2)} * p3^{\min(a3,b3)} * ----* pn^{\min(an,bn)}$$
$$LCM(a,b) = p1^{\max(a1,b1)} * p2^{\max(a2,b2)} * p3^{\max(a3,b3)} * ----* pn^{\max(an,bn)}$$
$$L..H.S = GCD(a,b) * LCM(a,b)$$
$$= [\, p1^{\min(a1,b1)} * p2^{\min(a2,b2)} * p3^{\min(a3,b3)} * ----* pn^{\min(an,bn)}\,] *$$
$$[p1^{\max(a1,b1)} * p2^{\max(a2,b2)} * p3^{\max(a3,b3)} * ----* pn^{\max(an,bn)}]$$
$$= [\, p1^{\min(a1,b1)} * p1^{\max(a1,b1)}] * [p2^{\min(a2,b3)} * p2^{\max(a2,b2)}] *$$
$$[p3^{\min(a3,b3)} * p3^{\max(a3,b3)}] * ----* [pn^{\min(an,bn)} * p2^{\max(an,bn)}]$$
$$= (p1^{a1} * p1^{b1}) * (p2^{a2} * p2^{b2}) * (p3^{a3} * p3^{b3}) * ----(pn^{an} * pn^{bn})$$
$$= (p1^{a1} * p2^{a2} * p3^{a3} * ----* pn^{an}) * (\, p1^{b1} * p2^{b2} * p3^{b3} * ----* pn^{bn}$$
$$form\, eq^{n}\ (1)$$
$$= a * b$$
$$= R.H.S$$
$$\therefore\ L.H.S = R.H.S$$
$$\therefore\ LCM(a,b).GCD(a,b) = a.b$$

Verification of this result for a =100, b = 80

100 = 1*2*2*5*5

80 = 1*2*2*2*2*5

GCD(100,80) = 1*2*2$A = \pi r^2$*5

= 20

LCM(100,80) = 1*2*2*2*2*5*5

$= 400$

$LCM(100,80) . GCD(100,80) = 100*80$

$20*400 = 8000$

$8000 = 8000$

OR

(C) Prove that :-

If A, B and C are Boolean matrices of compatible sizes, then

(AΘB) ΘC=AΘ(BΘC)

Solution:

Let us assume that

$A = [a_{ij}]\ m \times n$

$B = [\ b_{jk}]\ m\times n$

$C= [\ c_{kl}]\ m\times n$

$(A \vee B) \vee C = A\vee (B\vee C)$

i,e $(A+B)+C = A+(B+C)$

Now,

$A + B = [a_{ij}]\ m \mathbf{X}\ n + [b_{jk}]\ m\times n$

$= [a_{ij}] + b_{jk}]\ m\times n$

$(A + B) + C = [a_{ij}+ b_{jk}\ ]\ m\times n +[\ c_{kl}]\ m\times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m\times n \longrightarrow (1)$

$(B + C) = [b_{jk}]\ m \mathbf{X}\ n + [c_{kl}]\ m\times n$

$=[b_{jk} + c_{kl}]\ m\times n$

$(B + C\ ) + A=[a_{ij}]\ m\times n + [b_{jk} + c_{kl}]\ m\times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m\times n \longrightarrow (2)$

From equation (1) & (2)

We get $(A\vee B) \vee C= A\vee (B\vee C)$

i.e.

$$\boxed{(A\Theta B)\ \Theta C = A\Theta(B\ \Theta C)}$$

(D) Let m and n be integers. Prove that $n^2 = m^2$

If and only if n is m or n is –m.

2. EITHER

(A) Prove the equivalence.

$(P \vee Q) \wedge (7P \wedge Q)) \Leftrightarrow (7P \wedge Q)$ **without using truth table.**

Proof:- $(P \vee Q) \wedge (7P \wedge Q)) \Leftrightarrow (7P \wedge Q)$

L.H.S

$$\begin{array}{ll}
\Rightarrow (P \vee Q) \wedge (7P \wedge (7P \wedge Q)) & \\
\Rightarrow (P \vee Q) \wedge ((7P \wedge 7P) \wedge Q) & \{By\ associative\ properties \\
\Rightarrow (P \vee Q) \wedge (7P \wedge Q) & \{By\ idempotent\ properties \\
\Rightarrow (P \wedge (7P \wedge Q)) \vee (Q \wedge (7P \wedge Q)) & \{By\ distributive\ properties \\
\Rightarrow ((P \wedge 7P) \wedge Q) \vee ((Q \wedge Q) \wedge 7P) & \{By\ associative\ properties \\
\Rightarrow (F \wedge Q) \vee (Q \wedge 7P) & \{P \wedge 7P = F\ \&\ idempotent\ properties \\
\Rightarrow F \vee (Q \wedge 7P) & \{F \wedge Q = F \\
\Rightarrow F \vee (7P \wedge Q) & \{By\ cummutative\ properties \\
\Rightarrow (7P \wedge Q) & \{P \vee F = P \\
\Rightarrow R.H.S &
\end{array}$$

hence $\boxed{(P \vee Q) \wedge (7P \wedge Q)) \Leftrightarrow (7P \wedge Q)}$

(B)Show that the following premises are inconsistent:-

(i) If Jack misses many classes through illness,

then he fail high school.

(ii) If Jack fails high school, the he is uneducated.

(iii) If Jack reads a lot of books, then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books.

Proof:-

We have to prove that given premises are inconsistent. To prove inconsistent we have to derive contradiction from the given premises.

Let,E: Jack misses many classes.

S: Jack fails high School.

A: Jack read a lot of books.

H: Jack is uneducated.

Given premises are,

$$\begin{array}{l}
E \rightarrow S \\
S \rightarrow H \\
A \rightarrow 7H \\
E \rightarrow A
\end{array}$$

{1}	(1) $E \to S$	{Rule P
{2}	(2) $S \to H$	{Rule P
{1,2}	(3) $E \to H$	{Rule T i,e $P \to Q, Q \to R \Rightarrow P \to R$
{4}	(4) $A \to 7H$	{Rule P
{4}	(5) $H \to 7A$	{Rule T i,e $P \to Q \Leftrightarrow 7Q \to 7P$
{3,5}	(6) $E \to A$	{Rule T i,e $P \to Q, Q \to R \Rightarrow P \to R$
{6}	(7) $7E \vee 7A$	{Rlue T i,e $P \to Q \Rightarrow 7P \vee Q$
{7}	(8) $7(E \wedge A)$	{By Demorgans properties i,e $7E \vee 7A \Rightarrow 7(E \wedge A)$
{9}	(9) $E \wedge A$	{Rule P
{8,9}	(10) $7(E \wedge A) \wedge (E \wedge A)$	{Rule T i,e $P, Q \Rightarrow P \wedge Q$

There is conjunction implies a contradiction (FALSE); hence the given premises are inconsistent.

OR

(C) Define conjunctive normal form and obtain a conjunctive normal formof

$$7(P \vee Q) \leftrightarrow (P \wedge Q)$$

Conjunctive Normal Form:- Any formula which is equivalent to a given formula and which consist of product of elementary sum is called conjunctive normal form of given formula.

Proof:-

$7(P \vee Q) \leftrightarrow (P \wedge Q)$

$by\ R \leftrightarrow S \Leftrightarrow (R \to S) \wedge (S \to R)$

$\Leftrightarrow [7(P \vee Q) \to (P \wedge Q)] \wedge [(P \wedge Q) \to 7(P \vee Q)]$

$\Leftrightarrow [77(P \vee Q) \vee (P \wedge Q)] \wedge [7(P \wedge Q) \vee 7(P \vee Q)]$ $\{P \to Q \Rightarrow 7P \vee Q$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(7P \vee 7Q) \vee 7(P \vee Q)]$ $\{By\ Demorgans\ property\ \&\ 77P \Rightarrow P$

$\Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(7P \vee 7Q \vee 7P) \wedge (7P \vee 7Q \vee 7Q)]\{By\ Distributive\ property$

$\Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((7P \vee 7P) \vee 7Q) \wedge ((7Q \vee 7Q) \vee 7P)\{By\ Associative\ property$

$\Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (7P \vee 7Q) \wedge (7Q \vee 7P)$ $\{P \vee P = P$

$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (7P \vee 7Q) \wedge (7P \vee 7Q)$ $\{By\ commulative\ property$

$\Leftrightarrow (P \vee Q) \wedge (7P \vee 7Q)$ $\{P \wedge P = P$

It is the form of product of elementary sum of min terms.

Hence it is form of Principal Conjunction Normal Form.

(D)Show that R $\wedge$ (P $\vee$ Q) is a valid conclusion from the premises P $\vee$ Q ,

Q $\to$R , P $\to$M and 7 M.

Solution:

{1}	(1) 7 M	{Rule P
{2}	(2) P $\to$M	{ Rule P
{2}	(3) 7M $\to$7P	{ Rule T : P $\to$Q $\Leftrightarrow$ 7Q $\to$7P
{3}	(4) 7P	{ Rule T : P,P $\to$Q $\Leftrightarrow$ Q

{5}	(5) $P \vee Q$	{ Rule P
{5}	(6) $\neg P \rightarrow Q$	{Rule T : $\neg P \rightarrow Q \Leftrightarrow \neg\neg P \vee Q \Leftrightarrow P \vee Q$
{4,6}	(7) Q	{ Rule T : P, $P \rightarrow Q \Leftrightarrow Q$
{8}	(8)$Q \rightarrow R$	{ Rule P
{7,8}	(9) R	{ Rule T : Q,$Q \rightarrow R \Leftrightarrow R$
{9,5}	(10) $R \wedge (P \vee Q)$ {Rule T : P,Q $\Leftrightarrow P \wedge Q$	
	Hence Proved.	

3. EITHER

(A) Find an explicit formula for the sequence defined by

$a_n = 4a_{n-1} + 5a_{n-2}$ **with initial conditions** $a_1 = 2,\ a_2 = 6.$

Solution:

First find sequence for recurrence relation

$a_n = 4a_{n-1} + 5a_{n-2}$

For n = 3 $a_3 = 4a_{3-1} + 5a_{3-2}$

$= 4a_2 + 5a_1$

$= 4(6) + 5(2)$

$= 24 + 10$

$= 34$

For n = 4 $a_4 = 4a_{4-1} + 5a_{4-2}$

$= 4a_3 + 5a_2$

$= 4(34) + 5(6)$

$= 166$

For n=5 $a_5 = 4a_{5-1} + 5a_{5-2}$

$= 4a_4 + 5a_3$

$= 4(166) + 5(34)$

$=834$

$\therefore$ Sequence is 2,6,34,166,834----

The recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ is linear homogeneous

Equation of degree 2.

It associated equation is

$x^2 = 4x+5$

Rewriting this as

x^2 -4x-5=0

x^2 -5x+x-5=0

(x-5)(x+1)=0

X=5 or x=-1

The roots of the equation is s_1 =5 ands_2 =-1

Now, by teorem(i)

We can find value of u and v

From $a_n = us_1^n + vs_2^n$ -----------(A)

For n=1

$a_1 = us_1 + vs_{12}$

2 = u(5) + v(-1)

2 = 5u – v ------------(i)

For n = 2

$a_2 = us_1^n + vs_2^n$

6 = u$(5)^2$ + v $(-1)^2$

6= 25u+v -----------(ii)

Solving equation (i) and (ii)

5u – v = 2

25u + v = 6

\+ + +

30u = 8

$\boxed{U=8/30}$

Putting values of u in equation (i)

2 = 5 (8/30) – v

2 = (8/6) – v

2 = 8/6 – 2

2 = 8-12/6

V = -4/6

$\boxed{V = -2/3}$

Put value of u_1, v_1 , s_2 and s_2 in equation (A)

$a_n = us_1^n + vs_2^n$

a_n = (8/30) $(5)^n$ + (-2/3) $(-1)^n$

a_n = 8/30 $(5)^n$ + -2/3 $(-1)^n$

∴Which is required formula.

(B) Let A ={a,b,c,d} and let R be the relation on A that has the matrix.

$$M_r = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Draw the diagram of R. Find the indegree and outdegree of all vertices.

OR

(C) Let A = Z and let R = {(a, b)}∈ A X A : a ≡ r (mod 2) and b ≡ r (mod 2).

Show that the relation R is an equivalence relation.

(D) Prove that,

Let R be a relation on a set A. Then R^{∞} is the transitive Closure of R.

Solution:-

If a & b are in the set A then a R^{∞} iff there is a path in R from a to b.

Now R^{∞} is certainty transitive it a R^{∞} b and b R^{∞} c then the composition of path from a to b and b to c from sub path from a to b in R and so a R^{∞} c. To show that R^{∞} is the smallest transitive relation containing R.

We must show that if S is any transitive relation on A and R ⊆S then R^{∞} smallest of S($R^{\infty} \subseteq S$)

We know that if S is transitive then,

S^{∞} / S^{n} ⊆ S for all n. i,e if a & b are connected by a path of length n the a S b it follows that

$$S^{\infty} = \bigcup_{N \neq 1}^{\infty} aSb$$

$$S^{n} \ OR \ S^{\infty} = \bigcup_{n \neq 1}^{\infty} S^{n} \subseteq S$$

It is also true if R ⊆ S then R^{∞} is subset of S^{∞} since any path in R is also path in S patting this fact together, we see that,

If R ⊆ S and S is transitive on A then,

R^{∞} subset of S^{∞} ⊆S, R^{∞} ⊆ S

This means that R is the smallest of all transitive relation on A, that contains R^{∞}.

Hence theorem is proved.

4. EITHER

(A) Show that if n is a positive integer and $P^2 | n$, where p is prime number, then D_n is not Boolean algebra.

Proof:-

Suppose that $P^2 / n,$ $a / b \Rightarrow b = ac \;\; for\, sum\, c$

So, $n = p^2 . q$ for some positive integer q.

∵p is also a divisor of n and p is an element of Dn, (Divisor of n number i,e Dn)

→natural number i,e if Dn is a Boolean Algebra. Then p must have a complement p'.

Then GCD(p,p')=1

And LCM(P,P')=1

GCD.LCM(p,p')=p.p'

i,epp'=n

so, p'=n/p=1

p' =p.q

i,e GCD(p,pq) = 1

This is impossible.

∵p& p.q have p as a common divisor.

(B) Find the Hamiltonian circuit for the given graph.

Answer:

Hamiltonian Graph:

A Hamiltonian graph is a graph that on a Hamiltonian path.

A Hamiltonian path uses each vertex exactly once but edges be include.

OR

(C) Let L is a bounded distributive lattice. Prove that, if a complement of an element in L exists then it is unique.

Proof:-

Lattice suppose a' and a'' are two complements of an element a $\in L$

$$a \wedge a' = 0 \qquad a \wedge a'' = 0$$
$$a \vee a' = I \qquad a \vee a'' = I$$

We show that a'=a''

Now

$$a' = a' \wedge I$$
$$a' = a' \wedge (a \vee a'')$$
$$a' = (a' \wedge a) \vee (a' \wedge a'')$$
$$a' = 0 \vee (a' \wedge a'')$$
$$a' = a' \wedge a'' \qquad ------(1)$$

And

$$a'' = a'' \wedge I$$
$$a'' = a'' \wedge (a \vee a')$$
$$a'' = (a'' \wedge a) \vee (a'' \wedge a')$$
$$a'' = 0 \vee (a'' \wedge a')$$
$$a'' = a'' \wedge a'$$
$$a'' = a' \wedge a'' \qquad \{by\, commulative\ property \qquad ------(2)$$

$\therefore a' = a''$ (From equation (1) and (2))

$\therefore$ Complement if exists is unique.

$\therefore proved.$

(D) Prove that : -

A tree with n vertices has n-1 edges.

Proof:-

Consider tree T(V,E)

By using mathematical induction on the number of vertices, n in T.

Suppose, it is true n=m($\geq$ 2).

m- some positive integer.

To prove for n = m+1

Suppose, T has m+1 vertices.

If we remove an edge with end points u&v from T.

Then we are left with two sub trees T1(V1,E1) and T2(V2,E2)

Such that |V| = |V1| + |V2| AND |E| = |E1| + |E2|

T1 & T2 are connected with number cycles and having vertices less than n,

i,e |V1| $\leq$n & |V2| $\leq$ n,

i,e |E1| = |V1| -1; |E2| = |V2| -1

|V| = |V1| + |V2|

|V| = (|E1| + 1) + (|E2| + 1)

|V| = (|E1| + |E2| + 1) + 1

|V| = |E1| + 1

OR

|E| = (|V1| - 1) + (|V2| -1) + 1

|E| = (|V1| + |V2| - 1)

$|E| = m+1-1$

$|E| = m$

This is prove that T has m edges which is sequence number.

5. EITHER

(A) Define :-

(i) Semigroup

(ii) Monoid

(iii) Subsemigroup

(iv) Group homomorphism.

Answer:-

(i) Semigroup:-

Let S be a non-empty set and * be a binary operation on S. The algebraic system (S, *) is called a semigroup if the operation * is

(1) The operation * is a closed operation on set A.

(2) The operation * is an associative operation.

Or (S, *) is a semigroup if for any x, y, z ϵS,

$$\boxed{(x * y) * z = x * (y * z)}$$

Free semigroup:

If * is an associative binary operation, and (A,*) is a semigroup. The semigroup(A,*) is called free semigroup by A.

Ex:

Consider an algebraic system (S,*) where S = {1,2,3,5,7,9----} the set of all positive odd integers and * is a binary operation means multiplication. Determine whether (S,*) is a semigroup.

(ii) Monoid:-

Let us consider an algebraic system (M, *), where * is a binary operation on M. Then the system (M, *) is said to be a monoid if it satisfies the following properties:

(1) The operation * is a closure operation on set A.
(2) The operation * is an associative operation.
(3) There exists an identify element w. r. t. The operation *.

Ex:-

Consider an algebra system (N, +), where the set N = {0,1,2,3----} the set of natural numbers and + is an addition operation. Determine whether (N,+) is a monoid.

(iii) Subsemigroup:-

Let (S,*) be a semigroup and $T \subseteq S$, if the set T is closed number the operation * then (T,*) is said to be subsemigroups of (S,*).

Ex:

Consider a semigroup (N,+), where N is the sset of all natural number and + is an addition operation.

The algebric system (E,+) is a subsemigroup of (N,+), where E is a set of all +ve even integer.

(iv) Group homomorphism:-

Let (S,*) and (T,*) be two semigroups. An everywhere defined function f: $S \to T$ is called a homomorphism from (S,*) and (T,*)

If (a * b) = f(a) *f (b)

For all a and b in S.

If f is also onto.

We say that T is a homomorphic image of S.

(B) Define finite state machine. Construct digraph of machine whose table is

	a	b	c
S0	**S0**	**S0**	**S0**
S1	**S2**	**S3**	**S2**
S2	**S1**	**S0**	**S3**
S3	**S3**	**S2**	**S3**

Answer:-

Finite state machines:

Finite state machine that accepts more than one input and gives single output then it is Finite-State-Machine.

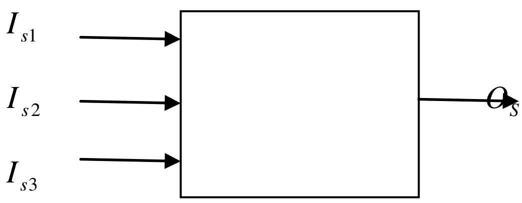


Fig: Finite-State Machine.

Where I_s = input signal, O_s = output signal

Here I_{s1}, I_{s2}, I_{s3} -------- I_{sn} is number of input which gives single output signal as shown in fig.

Definition:

Finite-State Machine define by 3-tuple (triple)

M = (S, I, F)

Where S = finite set of state of machine i,e S = $\{S_0, S_1, ----S_n\}$

I = finite set of input of machines.

F = Is the state transition function i,e F = $\{f_x / x \in I\}$

(for each $x \in I,$ a function $f_x : S \to S$)

Solution:

Let finite-state machines define by 3-tuple.

M = (S, I, F)

Where S = $\{S_0, S_1, S_2 S_3\}$

I = {a, b, c}

F = state transition function.

Given that: Transition Table

	a	**b**	**c**
S0	**S0**	**S0**	**S0**
S1	**S2**	**S3**	**S2**
S2	**S1**	**S0**	**S3**
S3	**S3**	**S2**	**S3**

From the above transition table draw digraph for the machine as follows.

Digraph:

OR

(C) Let G be the set of all non zero real numbers and Let a * b = ab/2 show that (G, *) is an abelian group.

Solution:-

If a, b are element in G the $\frac{ab}{2}$ is a non-zero real number.

To show: (G,*) is an abelian group.

Closure property:

The set G is closed under the operation *.

Since, a*b = $\frac{ab}{2}$ is a real number.

Hence, belongs to G.

Associative property:

The operation * is associative.

Let a, b, c ∈ G, then

We have

$$(a*b)*c = \left(\frac{ab}{2}\right)*c$$

$$= \frac{(ab)c}{4}$$

$$= \frac{abc}{4}$$

Similarly, a* (b*c) * a = $\left(\frac{ab}{2}\right)$

$$= \frac{a(bc)}{4}$$

$$= \frac{abc}{4}$$

Identity :

To find the identity element.

Suppose that 'e' is a +ve real number.

Then, e * a = a, where a$\in$ G

$$\frac{ea}{2} = a \qquad or \qquad e = 2$$

Similarly, a * e = a

$$\frac{ae}{2} = a \qquad or \qquad e = 2$$

Thus, the identity element in G is G.

Inverse :

Suppose that a $\in G$.

If $a^{-1} \in Q\, is$ an inverse of a, then a * $a^{-1} = 4$.

Therefore, $\frac{aa^{-1}}{4} = 4 \qquad or \qquad a^{-1} = \frac{4}{a}$

Thus, the inverse of element 'a' in G is $\frac{4}{a}$

Commutative :

The operation * on G is commutative.

Since, $a * b = \frac{ab}{2} = b * a$

Thus, the algebraic system (G, *) is closed, associative, identity element, inverse and commutative.

Hence, the system (G, *) is an abelian group.

(D) consider the semigroup (z, +) and the equivalence relation R on Z defined by aRb if and only if a $\equiv$ b (mod 2). Show that this relation is a congruence relation.

Solution:

Remember that if a $\equiv$ b (mod 2), then 2 | a – b.

We now show that this relation is a congruence relation as follows.

$a \equiv b \pmod 2$

and $c \equiv d \pmod 2$

then 2 divide $a - b$

and 2 divide $c - d$

so $a - b = 2m$

and $c - d = 2n$

where m and n are Z.

Adding, we have

$$(a - b) + (c - d) = 2m + 2n$$

Or $(a + c) - (b + d) = 2(m + n)$

So, $a + c \equiv b + d \pmod 2$

Hence, the relation is congruence relation.

EITHER

(A) Define:-

i. **Boolean Matrix**
ii. **Join of Boolean Matrices**
iii. **Meet of Boolean Matrices**
iv. **Boolean product and**

Show that $A \Theta (B \Theta C)=(A \Theta B) \Theta C$

If $A= \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{matrix}$ $B = \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{matrix}$ $C= \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$

Solution:

` Let us assume that

$A = [a_{ij}]\ m \times n$

$B = [\ b_{jk}]\ m\times n$

$C= [\ c_{kl}]\ m\times n$

$(A \vee B) \vee C = A\vee (B\vee C)$

i,e $(A+B)+C = A+(B+C)$

Now,

$A + B = [a_{ij}]\ m \text{X}\ n + [b_{jk}]\ m\times n$

$= [a_{ij}] + b_{jk}]\ m\times n$

$(A + B) + C = [a_{ij}+ b_{jk}\ ]\ m\times n +[\ c_{kl}]\ m\times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m\times n \longrightarrow$ (1)

$(B + C) = [b_{jk}]\ m \text{X}\ n + [c_{kl}]\ m\times n$

$=[b_{jk} + c_{kl}]\ m\times n$

$(B + C) + A=[\ a_{ij}]\ m\times n + [b_{jk} + c_{kl}]\ m\times n$

$= [a_{ij} + b_{jk} + c_{kl}]\ m\times n \longrightarrow$ (2)

From equation (1) & (2)

We get $(A\vee B) \vee C= A\vee (B\vee C)$

i.e.

$$\boxed{A \Theta (B \Theta C)=(A \Theta B) \Theta C}$$

(B) Make truth table for

i. **$(p \wedge q) \vee (ᒣp)$**

ii. **$(p \downarrow q) \downarrow r$**

Solution: **$(p \wedge q) \vee (\neg p)$**

Truth Table:

p	**q**	**$(p \wedge q)$**	**$(\neg p)$**	**$(p \wedge q) \vee (\neg p)$**
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

$(p \downarrow q) \downarrow r$

Truth Table:

p	**q**	**r**	**$(p \downarrow q)$**	**$(p \downarrow q) \downarrow r$**
T	T	T	F	F
T	T	F	F	T
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	T	F
F	F	F	T	F

OR

(C) **Use Induction method to prove that**

$$\overline{\left(\bigcap_{i=1}^{n} Ai\right)} = \bigcup_{i=1}^{n} \overline{Ai}$$

Solution:

$$\overline{A1 \cap A2 \cap A3 - - - \cap An} = \overline{A1} \cup \overline{A2} \cup \overline{A3} - - - \cup \overline{An}$$

Basic Steps:

Let n=1

P(A)= $\overline{A1} = \overline{A1}$

P(n) is true for n=1

Induction Steps:

Let us assume that P(n) is true for n=k

P(k) = $\overline{\left(\bigcap_{i=1}^{k} Ai\right)} = \bigcup_{i=1}^{k} \overline{Ai}$

$$\therefore \overline{A1 \cap A2 \cap A3 - - - \cap Ak} = \overline{A1} \cup \overline{A2} \cup \overline{A3} - - - \cup \overline{Ak} \longrightarrow \textbf{(1)}$$

Now,

We have to prove that P(n) is true for n=k+1

P(k+1) = $\left(\overline{\bigcap_{i=1}^{k+1} Ai}\right) = \bigcup_{i=1}^{k+1} \overline{Ai}$

$\overline{A1 \cap A2 \cap A3 --- \cap Ak+1} = \overline{A1} \cup \overline{A2} \cup \overline{A3} --- \cup \overline{Ak+1}$

L.H.S.

$\Rightarrow \overline{A1 \cap A2 \cap A3 --- \cap Ak+1}$

$\Rightarrow \overline{A1 \cap A2 \cap A3 --- Ak \cap Ak+1}$

$\Rightarrow \overline{A1 \cap A2 \cap A3 --- Ak} \cup \overline{Ak+1}$ {By demorgans property $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\Rightarrow \overline{A1} \cup \overline{A2} \cup \overline{A3} --- \cup \overline{Ak+1}$ {From eq. (1)

$\Rightarrow$ R.H.S.

Hence Proved

P(n) is true for n=k+1

(D) Let m and be integers. Prove that $n^2 = m^2$ If and if only if m=n or m= -n. Also prove that 3/(n^3 - n) for every positive integers

EITHER

Q.2

(A) Show that

$(P \vee Q) \wedge ¬(¬P \wedge (¬Q \vee ¬R)) \vee (¬P \wedge ¬Q) \vee ¬(P \wedge ¬R)$

Is tautology without using truth table

$\Rightarrow [(P \vee Q) ¬[¬P \wedge (¬Q \vee ¬R)]] \vee ¬[(¬P \wedge ¬Q) \vee ¬(P \wedge ¬R)]$ {by associative property

$\Rightarrow [(P \vee Q) \wedge ¬(¬P \wedge ¬(Q \vee R))] \vee [(¬P \wedge ¬Q \vee ¬R)]$ {By demorgans and distributive prop. Respt.

$\Rightarrow [(P \vee Q) \wedge ¬¬(P \vee (Q \wedge R))] \vee (¬P \wedge ¬(Q \wedge R))$ {bydemorgans property

$\Rightarrow [(P \vee Q) \wedge (P \vee (Q \wedge R)] \vee ¬(P \vee (Q \wedge R)$ { by demorgans property & $¬¬A \Rightarrow$ A

$\Rightarrow [P \vee (Q \wedge (Q \wedge R))] \vee ¬(P \vee (Q \wedge R)]$ {by distributive property.

$\Rightarrow [P \vee (Q \wedge Q) \wedge R) \vee ¬(P \vee Q \wedge R)]$ {by associative property

$\Rightarrow (P \vee (Q \wedge R)) \vee ¬(P \vee (Q \wedge R))$ {by idempotent property & $P \wedge P \Rightarrow P$

$\Rightarrow$ T {$P \vee ¬P \Rightarrow T$

Hence Proved

(B)Define conjunctive normal form and obtain a conjunctive normal formof

$$7(P \vee Q) \leftrightarrow (P \wedge Q)$$

Conjunctive Normal Form:- Any formula which is equivalent to a given formula and which consist of product of elementary sum is called conjunctive normal form of given formula.

Proof:-

$7(P \vee Q) \leftrightarrow (P \wedge Q)$

$by\, R \leftrightarrow S \Leftrightarrow (R \rightarrow S) \wedge (S \rightarrow R)$

$\Leftrightarrow [7(P \vee Q) \rightarrow (P \wedge Q)] \wedge [(P \wedge Q) \rightarrow 7(P \vee Q)]$

$\Leftrightarrow [77(P \vee Q) \vee (P \wedge Q)] \wedge [7(P \wedge Q) \vee 7(P \vee Q)] \quad \{P \rightarrow Q \Rightarrow 7P \vee Q$

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(7P \vee 7Q) \vee 7(P \vee Q)] \quad \{By\ Demorgans\ property\ \&\ 77P \Rightarrow P$

$\Leftrightarrow [(P \vee Q \vee P) \wedge (P \vee Q \vee Q)] \wedge [(7P \vee 7Q \vee 7P) \wedge (7P \vee 7Q \vee 7Q)] \{By\ Distributive\ property$

$\Leftrightarrow ((P \vee P) \vee Q) \wedge ((Q \vee Q) \vee P) \wedge ((7P \vee 7P) \vee 7Q) \wedge ((7Q \vee 7Q) \vee 7P) \{By\ Associative\ property$

$\Leftrightarrow (P \vee Q) \wedge (Q \vee P) \wedge (7P \vee 7Q) \wedge (7Q \vee 7P) \quad \{P \vee P = P$

$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (7P \vee 7Q) \wedge (7P \vee 7Q) \quad \{By\ commulative\ property$

$\Leftrightarrow (P \vee Q) \wedge (7P \vee 7Q) \quad \{P \wedge P = P$

It is the form of product of elementary sum of min terms.

Hence it is form of Principal Conjunction Normal Form.

(C) Show that R $\wedge$ (P $\vee$ Q) is a valid conclusion from the premises P $\vee$ Q , Q $\rightarrow$ R , P $\rightarrow$ M and $\neg$ M.

Solution:

	{1}	(1) $\neg$M	{Rule P
	{2}	(2) P $\rightarrow$ M	{ Rule P
$\neg A$ π		(3) $\neg$M - $\neg$P	{ Rule T : P $\rightarrow$ Q $\Leftrightarrow$ $\neg$Q – $\neg$P
	{3}	(4) $\neg$P	{ Rule T : P $\rightarrow$ Q $\Leftrightarrow$ Q
		(5) P $\vee$ Q	{ Rule P
$= r^2$		(6) $\neg$P $\rightarrow$ Q	{Rule T : $\neg$P $\rightarrow$ Q $\Leftrightarrow$ $\neg\neg$P $\vee$ Q $\Leftrightarrow$ P $\vee$ Q
	{4,6}	(7) Q	{ Rule T : P $\rightarrow$ Q $\Leftrightarrow$ Q
	{8}	(8)Q $\rightarrow$ R	{ Rule P
	{7,8}	(9) R	{ Q,Q $\rightarrow$ R $\Leftrightarrow$ R
	{9,5}	(10) R $\wedge$ (P $\vee$ Q)	{Rule T : P,Q $\Leftrightarrow$ P $\wedge$ Q

Hence Proved.

(D) Show that

(x) (P(x) $\rightarrow$ Q(x)) $\wedge$ (x)(Q(x) $\rightarrow$ R(x)) $\Rightarrow$ (x)(P(x) $\rightarrow$ R(x))

Solution:

Given Premises are

$(x)\,(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x))$

We have to derives,

$(x)(P(x) \rightarrow R(x))$

{1}	(1) $(x)(P(x) \rightarrow Q(x))$	{Rule P
{2}	(2) $P(y) \rightarrow Q(y)$	{Rule US : $(x)A(x) \rightarrow A(y)$
{3}	(3) $(x)(Q(x) \rightarrow R(x))$	{Rule P
{3}	(4) $Q(y) \rightarrow R(y)$	{Rule US : $(x)A(x) \rightarrow A(y)$
{2,4}	(5) $P(y) \rightarrow R(y)$	{Rule T : $P \rightarrow Q,\ Q \rightarrow R \Rightarrow P \rightarrow P$
{5}	(6) $(x)(P(x) \rightarrow R(x))$	{Rule UG : $A(y) \rightarrow x\ A(x)$

Hence Prove.

Q.3 EITHER

(A) Define Cartesian product of two sets, partition of a set and prove that $A\times(B \cup C)=(A\times B) \cup (A\times C)$

Solution:

Cartesian product of two sets:

If A and B are the two non-empty sets,we define the product set or Cartesian product A×B as the set of all ordrded pair(a,b) with $a \in A$ and $b \in B$.

Thus.

$$\boxed{A\times B = \{(a,b) \mid a \in A \text{ and } b \in B\}}$$

Ex: let A={ 1,2,3} and B= {r,s}. Determine the product set of A×B and B×A.

Solution: Let A={1,2,3} and B={r,s}

To find : (1) A×B (2) B×A

(1) The Cartesian product of A and B is
A×B={(1,r), (1,s),(2,r),(2,s),(3,r),(3,s) }

(2) The Cartesian product or product sets of B and A is
B×A ={(r,1),(r,2),(r,3),(s,1),(s,2),(s,3)}

Prove that $A\times(B \cup C)=(A\times B) \cup (A\times C)$

Solution : Let $(x,y) \in A \times (B \cup C) \Rightarrow x \in A$ and $y \in B \cup C$

$\Rightarrow X \in A \text{ and } (y \in B \text{ or } y \in C)$

$\Rightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)$

$\Rightarrow (x,y) \in A\times B$ or $(x,y) \in (A\times C)$

$\Rightarrow (x,y) \in (A\times B) \cup (A\times C)$

Therefore, $A\times(B\cup C) \subset (A\times B)\cup(A\times C)$……..(1)

Now,

Conversely

Let $(x,y) \in (A\times B)\cup(A\times C)$

$\Rightarrow (x,y) \in (A\times B)$ or $(x,y) \in (A\times C)$

$\Rightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)$

$\Rightarrow x \in A \text{ and } y \in B \text{ or } B \text{ or } y \in C$

$\Rightarrow x \in a \text{ and } y \in (B\cup C)$

Therefore, $(A\times B)\cup(A\times C) \subset A\times(B\cup C)$**…….(2)**

From (1) and (2) , we have

$A\times(B\cup C) = (A\times B)\cup(A\times C)$

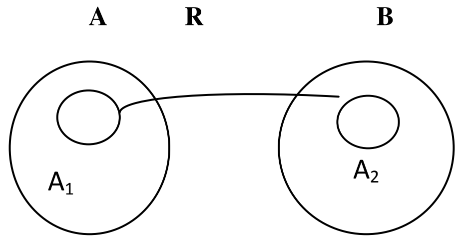
Hence proved.

(B) Let R is a relation from A to B, and let A_1 and A_2 be subsets of A. Then show that

(1) $R(A_1\cup A_2)= R(A_1)\cup R(A_2)$ and

(2) $R(A_1\cap A_2)= R(A_1)\cap R(A_2)$

Solution:



(1) $R(A_1\cup A_2)= R(A_1)\cup R(A_2)$

Let $y \in R(A_1\cup A_2)$

$\Rightarrow \exists\, x \in A_1\cup A_2$ s.t. $(x,y) \in R$ or xRy

$\Rightarrow \exists\, x \in A_1$ or $x \in A_2$ or $x \in A_1$ & A_2 s.t $(x,y) \in R$

If $x \in A_1$ s.t $(x,y) \in R$ then $y \in R(A_1)$

And If $x \in A_2$ s.t $(x,y) \in R$ then $y \in R(A_2)$

Now,

$y \in R(A_1)$ or $y \in R(A_2) \Rightarrow y \in R(A_1)\cup(A_2)$

$\therefore R(A_1 \cup A_2) \subseteq R(A_1)$ or (A_2) …….(1)

Now, suppose,

$y \in R(A_1) \cup R(A_2)$

$\Rightarrow y \in R(A_1)$ or $y \in R(A_2)$ or $y \in R(A_1)$ & $R(A_2)$

If $y \in R(A_1)$ then $\exists\, x \in A_1$ s.t. $(x,y) \in R$

If $y \in R(A_2)$ then $\exists\, x \in A_2$ s.t $(x,y) \in R$

$\therefore$ we have

$\exists\, x \in A_1$ or A_2 s.t $(x,y) \in R$

i.e. $\exists\, x \in A_1 \cup R(A_2) \Rightarrow (x,y) \in R \Rightarrow y \in R(A_1 \cup A_2)$

$\therefore R(A_1) \cup R(A_2) \subseteq R(A_1 \cup A_2)$…….(2)

From (1) & (2) we get

$$\boxed{\mathbf{R(A_1 \cup A_2) = R(A_1) \cup R(A_2)}}$$

Hence proved

(2) $R(A_1 \cap A_2) = R(A_1) \cap R(A_2)$

Let $y \in R(A_1 \cap A_2)$

$\Rightarrow \exists\, x \in A_1 \cap A_2$ s.t. $(x,y) \in R$

$\Rightarrow \exists\, x \in A_1$ and $x \in A_2$ s.t $(x,y) \in R$

Now,

$x \in A_1$ and $(x,y) \in R \Rightarrow y \in R(A_1)$ and $x \in A_2$ and $(x,y) \in R \Rightarrow y \in R(A_2)$

$\therefore y \in R(A_1)$ & $y \in R(A_2) \Rightarrow y \in R(A_1) \cap y \in R(A_2)$

$$\boxed{R(A_1 \cap A_2) = R(A_1) \cap R(A_2)}$$

(C) Let a={1,2,3} and let the relation R and S on A are

R={(1,1), (1,2), (2,1), (1,3), (3,1)}

S={(1,1), (1,2), (2,1), (2,2), (3,3)}

Find $\overline{R}$, R^{-1}, $\overline{S}$, S^{-1}, $S \cap S, R \cup S$

(D)Let A be set with |A|= n and Let R be arelation on A then prove that R^{∞} **=R** $\cup$ **R²** $\cup$ **------** $\cup$ **Rⁿ**

Q.4 EITHER

(A) Let the number of edges of G be M. Then prove that G has a Hamiltonian circuit if

$$m \geq \frac{1}{2}(n^2 - 3n + 6)$$

Proof:

Suppose U& V are two vertices of graph G that are not adjacent.

Let H be a graph product by elementary vertices U & K from G.

$\therefore$ H has n-2 vertices

$\therefore$ no. **of** edges in H are m –degree of a- degree of V.

$\therefore$ maximum no. of edges in H are n-2!

$$\frac{(n-2)!}{2!(n-2-2)!}$$

$$\Rightarrow \frac{(n-2)!}{2!(n-4)!}$$

$$\Rightarrow \frac{(n-2)(n-3)(n-4)!}{2!(n-4)!}$$

(n-4)! Get Cancel

$$\Rightarrow \frac{(n-2)(n-3)}{2}$$

$$\Rightarrow \frac{1}{2}(n^2 - 5n + 6)$$

$$\therefore m - deg(U) - deg(V) \leq \frac{1}{2}(n^2 - 5n + 6)$$

$$m - \frac{1}{2}(n^2 - 5n + 6) \leq m - deg(U) - deg(V)$$

or

$$deg(U) + deg(V) \geq m - \frac{1}{2}(n^2 - 5n + 6)$$

$$deg(U) + deg(V) \geq \frac{1}{2}(n^2 - 3n + 6) - \frac{1}{2}(n^2 - 5n + 6)$$

$$deg(U) + deg(V) \geq \frac{1}{2}[n^2 - 3n + 6 - n^2 - 5n - 6]$$

n^2 &- n^2 and +6 & -6 get cancle

deg(U)+deg(V) $\geq \frac{1}{2} \times 2n$

deg(U)+deg(V) $\geq$ n

$\therefore$ given graph has Hamiltonian Circuit

Hence theorem is proved.

(B) Define partial order set, chain , lexicographic , Isomorphism and show that the function f:A $\rightarrow$ A' define by f(a)=2(a) is an isomorphism from (A≤) to

(A'≤) where A is a set of positive integers , A' is a set of positive even integers.

Solution:

partial order set :

Let A is a relation or set A .then relation R is called partial order. If it is reflexive, antisymentric and transitive.

If R is a partial order relation on set A. then set A together with partial order relation R is know as partial orderd set or partial order set.

Ex. Let Z be a set f integers "≤" be a relation on Z.

$\therefore$ Reflexive property is satisfied .

($\therefore$ a≤a $\forall$ a$\in$ z)

Let a,b$\in$ z

a≤ b and b ≤ a $\Rightarrow$ a=b

$\therefore$ Antisymmentric property is satisfied

a≤ b and b ≤ c $\Rightarrow$ a≤ c

$\therefore$ Transitive property is satiesfide.

$\therefore$"≤" is a partial oreder relation on Z

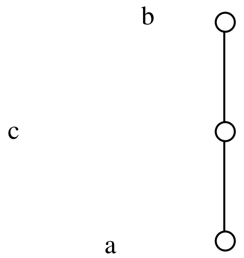
Similarly "≥"is also a partial oreder relation on Z.

Chain order set:

If every pair of element in a poset is comparable than poset A is called linear order set .Or set A is chain .

Ex. A={a,b,c}

a ≤ c, c ≤ b



This order is in linear or chain.

Hence it is called chain or linear orderd.

Lexicographic:

Let A×B is a cartesion product of two sets A &B .we define "<"as follow.

(a b) < (a' b') if a< a' or if a=a' then b< b'

This is used in dictionary.

Hence it is also as dictionary

Ex. Help, help

Help< help

Isomorphism:

Let (A≤) and (A'≤') be posets and let f: A $\rightarrow$ A!be a one-to-one correspondence between f: A & A! The function f is called an Isomorphism from A to A'

It for any a, b $\in$ A, a≤ b

$\Leftrightarrow$ f(a) ≤' a(b).

(Proof left)

(c) Define bounded lattice, distributives lattice, complemented lattice, modular lattice and prove that if L is a bounded distributive lattice then if complement exists, it is unique.

Solution:

(D) For Boolean polynomial

P(x,y,z)=$(x \wedge y) \vee (y \wedge z)$

Contrast truth table and show the polynomial by logic diagram.