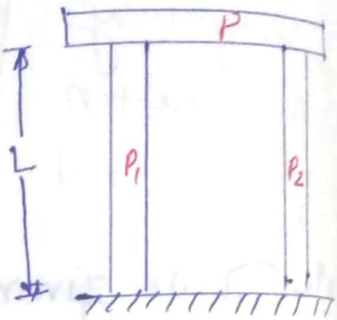
# *). Stress and strain in composite members.

**Composite member :→**

If two or more members of different materials are connected together and are subjected to load, the the combination is called composite member.

Consider a composite bar of length L made up of two different material connected rigidly together in parallel and carrying a compressive load 'p' as shown in fig.




let $P_1$ = load shared by bar 1,
$P_2$ = load shared by bar 2'
$P$ = Total load on bar.

$\therefore \quad P = P_1 + P_2$

$\therefore \quad P = \sigma_1 A_1 + \sigma_2 A_2 \qquad (\because \sigma = \frac{P}{A})$

let $\delta l$ = decrease in length of the bar.

For a composite bar, decrease or increase in length of each ~~par~~ bar must be equal i.e strain in each bar must be equal,

$\therefore$ strain in bar 1, $e_1 = \frac{\sigma_1}{E_1} \qquad (\because E_1 = \frac{\sigma_1}{e_1})$

Strain in bar 2, $e_2 = \frac{\sigma_2}{E_2}$

Now $e_1 = e_2$

$\therefore \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$

$\sigma_1 = \frac{E_1}{E_2} \sigma_2$

$\boxed{\sigma_1 = m \, \sigma_2}$

$(\because \frac{E_1}{E_2} = m =$ modular ratio)

Examples based on composite bar.



1). A steel tube 40 mm inside diameter and 4 mm metal thickness is filled with concrete. Determine the stress in each material due to an axial thrust of 60 kN.

Take $E_s = 2.1 \times 10^5$ N/mm²

$E_c = 0.14 \times 10^5$ N/mm².

Data given :->

$d = 40$ mm $\qquad P = 60$ kN $= 60 \times 10^3$ N.

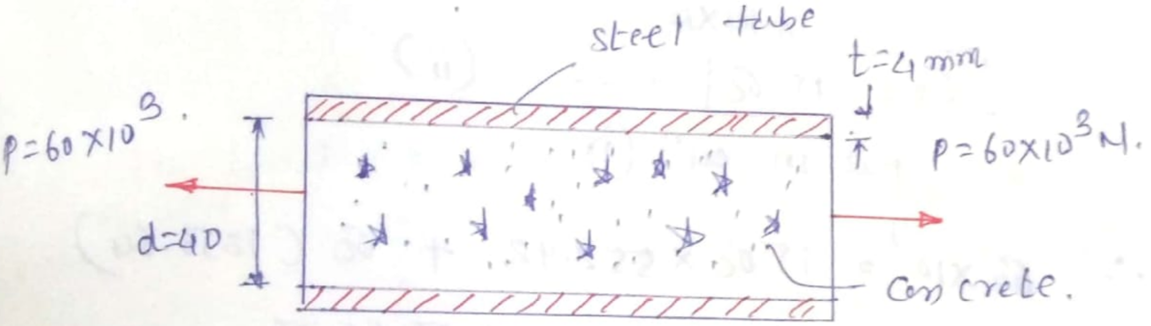
$t = 4$ mm.

$E_s = 2.1 \times 10^5$ N/mm²

$E_c = 0.14 \times 10^5$ N/mm²

To find $\sigma_c = ?$

$\sigma_s = ?$




Solⁿ :-> Outside diameter of the steel tube.

$D = d + 2t$

$= 40 + 2(4)$

$D = 48$ mm.

$\therefore$ Area of steel tube $A_s = \frac{\pi}{4}(D^2 - d^2)$

$A_s = \frac{\pi}{4}(48^2 - 40^2)$

$A_s = 552.9216$ mm².

Area of concrete $A_c = \frac{\pi}{4} d^2$

$= \frac{\pi}{4}(40)^2 = 1256.64$ mm².

∴ using equation.
Total load
$$P = P_1 + P_2$$
$$P = \sigma_1 A_1 + \sigma_2 A_2$$ we have
$$P = \sigma_s A_s + \sigma_c A_c$$
$$60 \times 10^3 = \sigma_s \times 552.92 + \sigma_c (1256.64) \quad \text{---(i)}$$

using the relation.
$$\sigma_1 = \frac{E_1}{E_2} \sigma_2$$ we have
$$\therefore \sigma_s = \frac{E_s}{E_c} \sigma_c$$
$$\therefore \sigma_s = \frac{2.1 \times 10^5}{0.14 \times 10^5} \times \sigma_c$$
$$\boxed{\sigma_s = 15 \sigma_c} \quad \text{----- (ii)}$$

put in eq$^n$ (i)
$$\therefore 60 \times 10^3 = 15 \sigma_c \times 552.92 + \sigma_c (1256.64)$$
$$= 8293.8 \sigma_c + 1256.64 \sigma_c$$
$$60 \times 10^3 = 9550.64 \sigma_c$$
$$\therefore \sigma_c = \frac{60 \times 10^3}{9550.44}$$
$$\boxed{\sigma_c = 6.28 \ N/mm^2}$$

put in eq$^n$ (2)
$$\Rightarrow \sigma_s = 15 \times \sigma_c$$
$$= 15 \times 6.28$$
$$\boxed{\sigma_s = 94.23 \ N/mm^2}$$

2) A concrete column of cross-sectional area 400 X 400 mm is reinforced by four longitudinal 50 mm diameter round steel bars placed at each corner. If the column carries a compressive load of 300 kN. determine

i) Load carried.

ii) The compressive stress produced in the concrete and steel bars.

Young's Modulus of Elasticity of Steel is 15 times that of ~~steel~~. Concrete.



Data given.

Area of concrete column = 400 X 400 mm.

diameter of steel bar d = 50 mm.

No. of bars = 4.

$P = 300\ kN = 300 \times 10^3\ N.$

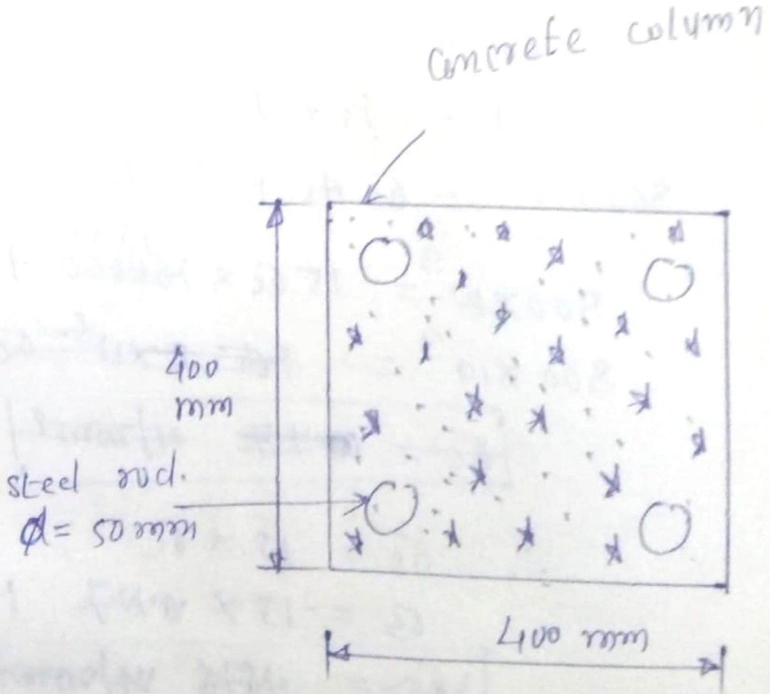
$E_s = 15\ E_c.$

To find $P_s = ?$

$P_c = ?$

$\sigma_c = ?$

$\sigma_s = ?$




Soln :→

Area of concrete

column = 400 X 400

$A = 160000\ mm^2.$

Area of steel

rod $(A_s) = 4 \times \frac{\pi}{4} d^2$

$= 4 \times \frac{\pi}{4} \times (50)^2$

$A_s = 7850\ mm^2.$

$\therefore$ Remaining Area of concrete

$A_c = A - A_s$

$A_c = 160000 - 7850$

$A_c = 152150 \ mm^2$.

As ~~Es = 15 Ec~~ (given).

As we know

$e_s = e_c$

$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$

$\therefore \sigma_s = \frac{E_s}{E_c} \sigma_c$

$\boxed{\sigma_s = 15 \sigma_c}$ $(\because E_s = 15 E_c)$

Now Total load shared by column

$P = P_s + P_c$

$300 \times 10^3 = \sigma_s A_s + \sigma_c A_c$

$300 \times 10^3 = 15 \sigma_c \times \cancel{160000}\ 7850 + \sigma_c (152150)$

$300 \times 10^3 = \cancel{2552 \times 10^6\ \sigma_c}\ 269.9 \times 10^3$

~~$\sigma_c = 0.117$ N/mm²~~ $\boxed{\sigma_c = 1.111 \ N/mm^2}$

$\therefore \sigma_s = 15 \times \sigma_c$

$\sigma_s = 15 \times \cancel{0.117}\ 1.111$

~~$\sigma_s = 1.678$ N/mm²~~ $\boxed{\sigma_s = 16.67 \ N/mm^2}$

~~Now~~

~~$P_s = \sigma_s \times A_s$~~

~~$= 1.677 \times 160000$~~

~~$P_s = 268.27 \times 10^3$ N~~

~~$P_c = \sigma_c \times A_c$~~

~~$P_c = 0.117 \times 152150$~~

~~$P_c = 17.801 \times 10^3$ N.~~

Now load of steel rod

$P_s = \sigma_s \times A_s$

$= 16.67 \times 7850$

$\boxed{P_s = 130.88 \times 10^3 \ N}$

and

load on concrete

$P_c = \sigma_c \times A_c$

$= 1.111 \times 152150$

$\boxed{P_c = 169.03 \times 10^3 \ N}$

(3) A copper rod of 40 mm diameter is surrounded tightly by cast-iron tube of 80 mm external dia. the end being firmly fastened together. When put to a compressive load of 30 KN, what load will be shared by each? Also determine the amount by which the compound bar shortens if it is 2 m long. Take $E_{ci} = 175 \ GN/m^2$
$E_c = 75 \ GN/m^2$)

(Ans $P_{ci} = 26.25$ KN

$P_c = 3.75$ KN.

$\delta l = 0.0796$ mm)

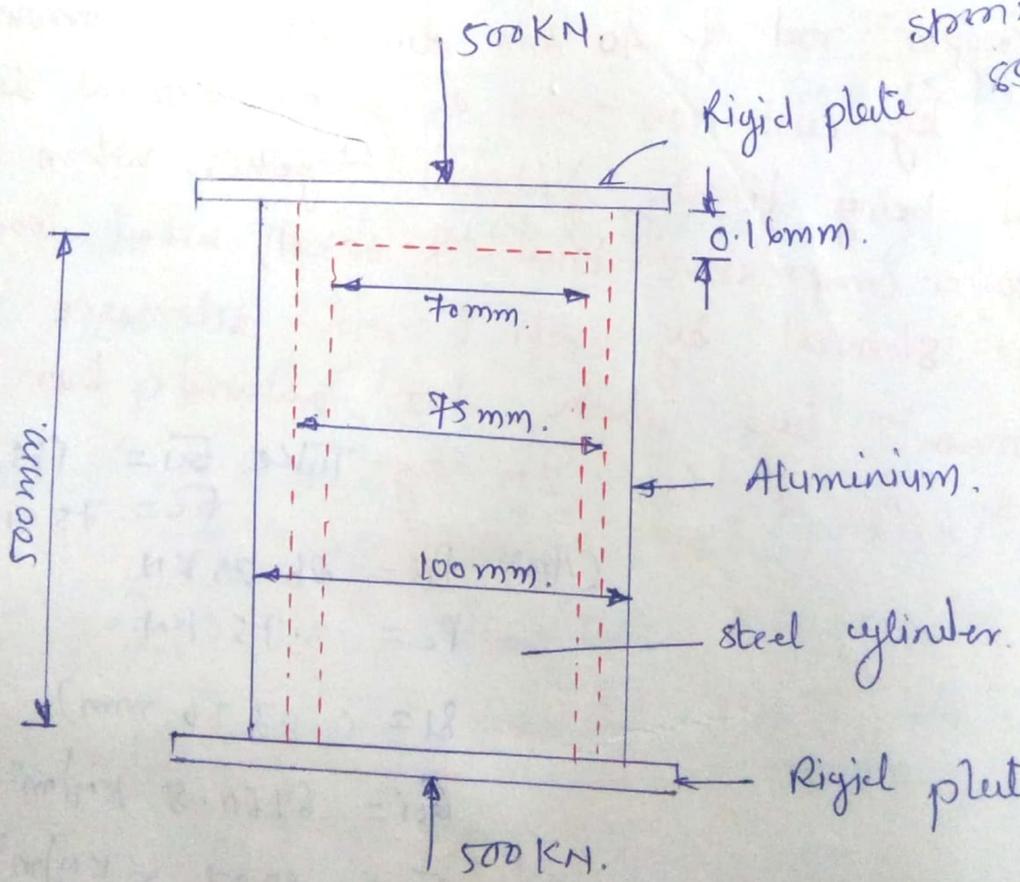
$\sigma_{ci} = 6960.8 \ KN/m^2$

$\sigma_c = 2987.5 \ KN/m^2$)

Imp 4) A solid steel cylinder 500 mm long and 70 mm diameter is placed inside an aluminium cylinder having 75 mm inside and 100 mm outside dia. The aluminium cylinder is 0.16 mm longer than the steel cylinder. An axial load of 500 kN is applied to the bar and cylinder through rigid cover plates as shown in fig. Find the stresses developed in the steel cylinder and aluminium tube. Assume for steel $E = 220\ GN/m^2$ and for aluminium $E = 70\ G$[illegible]

(Ans Total stress for aluminium 27.24 + 22.23

stress in steel 85.61 MN/m²)



since the aluminium cylinder is 0.16 mm longer than the steel cylinder. the load required to compress this cylinder by 0.16 mm will be calculated as

$$\delta l = \frac{Pl}{AE}$$

$$0.16 = \frac{P \times 500.16}{\frac{\pi}{4}(100^2 - 75^2) \times 70 \times 10^3}$$

$$P = 76944.57 \text{ N.}$$

When the aluminium cylinder is compressed by its extra length 0.16mm., the load then shared by both aluminium as well as steel cylinder. will be

~~400000~~ $500000 - 76944.57 = 423055.4$ N.

let $e_s$ = strain in steel cylinder
$e_a$ = strain in Aluminium cylinder.

As both The cylinder as of the same length then.

$$e_s = e_a$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a}.$$

$$\sigma_s = \frac{E_s}{E_a}\sigma_a.$$

$$\sigma_s = \frac{220 \times 10^3}{70 \times 10^3}\sigma_a$$

$$\boxed{\sigma_s = 3.142\,\sigma_a}$$

Also

$P = P_s + P_a$.

$= \sigma_s A_s + \sigma_a A_a$.

$423055.4 =$

$423055.4 = 3.142\ \sigma_a \times \frac{\pi}{4}(70)^2 + \sigma_a \frac{\pi}{4}(100^2 - 7$

$$\boxed{\sigma_a = 27.23\ N/mm^2}$$

$\sigma_s = 3.142 \times 27.23$

$$\boxed{\sigma_s = 85.58\ N/mm^2}$$

Now stress in the aluminium cylinder due to load 76944 N.

$= \dfrac{76944}{\frac{\pi}{4}(100^2 - 75^2)}$

$= 22.39\ N/mm^2$

$\therefore$ Total stress in aluminium cylinder

$= 27.23 + 22.39$

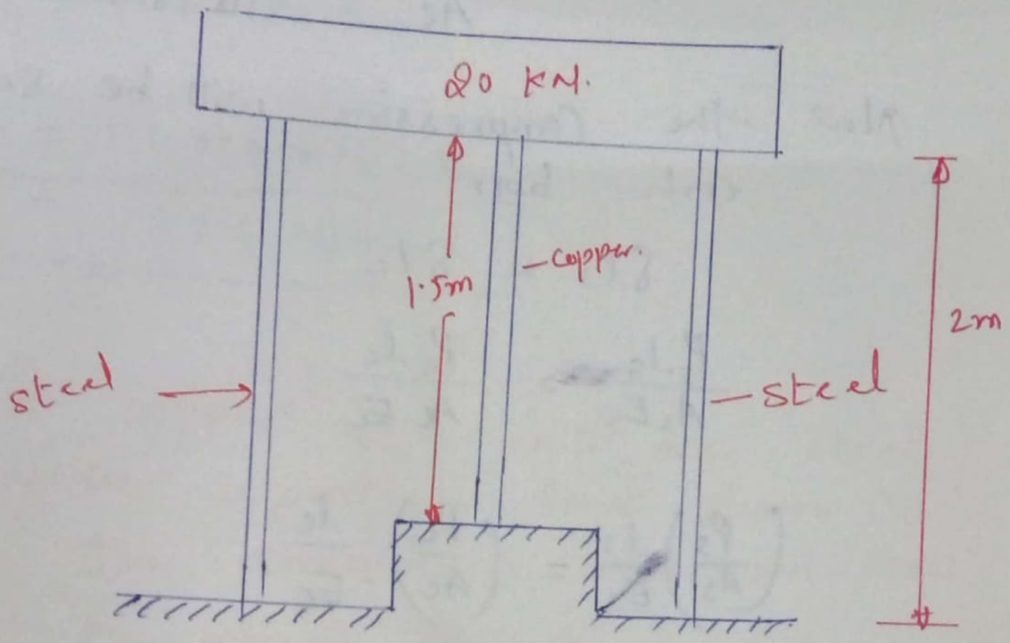
$= 49.62\ N/mm^2$

Ans

~~stress in steel~~

(81)

Two steel rods and one copper rod each of 20 mm ϕ together support a load of 20 KN as shown in fig. find the stresses in the rods. $E_s = 210 GPa$ and $E_c = 110 GPa$.




Data given. $d_s = 20 mm$.
$d_c = 20 mm$.
$l_s = 2 m = 2000 mm$
$l_c = 2 m = 2000 mm$.
$P = 20 KN$.
$E_s = 210 GPa = 210 \times 10^3 N/mm^2$
$E_c = 110 GPa = 110 \times 10^3 N/mm^2$.

To find $\sigma_s = ?$
$\sigma_c = ?$

Soln: → ~~Compression of each bar will~~ ~~be same.~~
∴ ~~$\delta l_s = \delta l_c$~~

Area of steel $= \frac{\pi}{4}(d_s)^2$

$A_s = 314.16 \text{ mm}^2$

Area of copper $= \frac{\pi}{4}(d_c)^2$

$A_c = 314.16 \text{ mm}^2$

Also the compression will be same for each bar

$$\delta l_s = \delta l_c$$

$$\frac{P_s l_s}{A_s E_s} = \frac{P_c l_c}{A_c E_c}$$

$$\left(\frac{P_s}{A_s}\right)\frac{l_s}{E_s} = \left(\frac{P_c}{A_c}\right)\frac{l_c}{E_c}$$

$$\sigma_s \frac{l_s}{E_s} = \sigma_c \frac{l_c}{E_c}$$

$$\sigma_s \times \frac{2000}{210 \times 10^3} = \sigma_c \times \frac{1500}{110 \times 10^3}$$

$$\boxed{\sigma_s = \cancel{1.4813}\ 1.4318\ \sigma_c}$$

Now the total load shared by two steel rod and one copper rod.

$$P = 2P_s + P_c$$

$$20 \times 10^3 = 2\sigma_s A_s + \sigma_c A_c$$

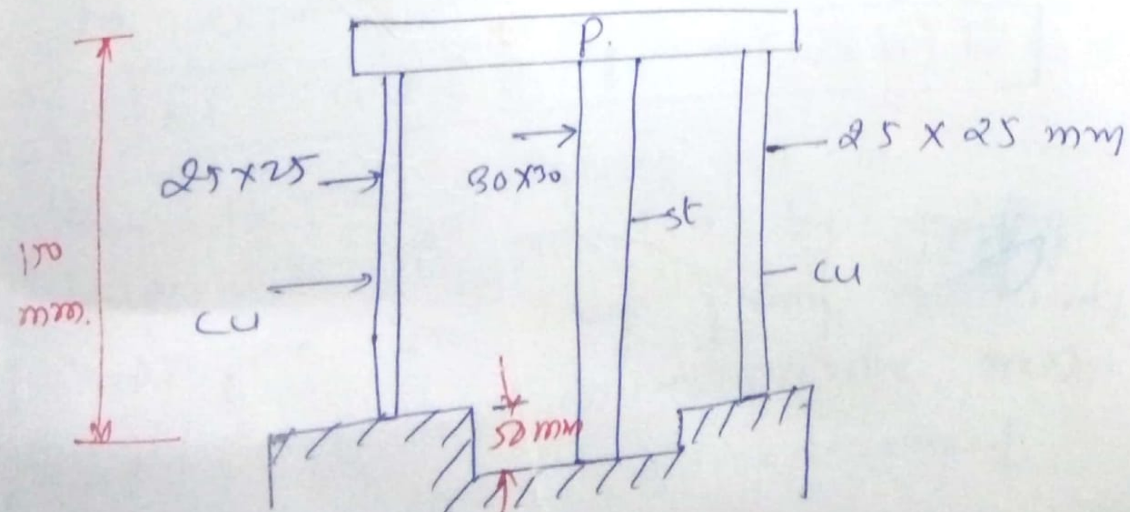
$$20\times10^3 = 2\times1.4318\,\sigma_c \times 314.16 + \sigma_c \times 314.16$$
$$= 899.63\,\sigma_c + 314.16\,\sigma_c$$
$$20\times10^3 = 1213.79\,\sigma_c$$

$$\sigma_c = 16.48\ N/mm^2$$

$$\therefore\ \sigma_s = 1.4318\times16.48$$
$$\boxed{\sigma_s = 23.6\ N/mm^2}$$

H.W 6) Two copper rods and one steel rod together support a load shown in fig. If the stresses in copper and steel are not to exceed $60\times10^6\ N/m^2$ and $120\times10^6\ N/m^2$ respectively, find the safe load that can be supported. Young's modulus for steel is twice that of copper.

(Ans $P = 156000\ N$).



## Thermal or Temperature stress and strain. (33)

Whenever there is increase or decrease in the temperature of a body, there is corresponding increase or decrease in its dimension. When a body is free to expand or contract due to rise or fall of the temperature, no stresses are induced in the body. But if this expansion or contraction due to temperature variation is wholly or partially prevented by application of external forces, some stresses are induced in the body. Such stresses are ~~known~~ known temperature stresses and corresponding strain due to temperature stresses are called the temperature strain.

Let the body of uniform section and L be the length and heated through $t°C$. The length of body will increase depending upon its coefficient of linear expansion $\alpha$. The increase in length due to increase in temperature when the body is free to expand will be

$$\boxed{\delta l = \alpha t L.} \quad \boxed{\delta l = \alpha (t_2 - t_1) L}$$

Now, If this expansion due to increase in temperature is prevented by applying external compressive force by fixing the body to rigid support, then compressive stress and compressive strain will developed.

∴ Compressive strain $e = \frac{\text{change in length}}{\text{original length}}$

$$e = \frac{\delta l}{L}$$

$$e = \frac{\alpha t L}{L}$$

$\boxed{e = \alpha t}$ or temperature strain

∴ From Hooke's Law

$$E = \frac{\sigma}{e}$$

∴ compressive stress $= e \times E$

$\boxed{\sigma = \alpha t E}$ or Temperature stress.

( Conversely, if the body is subjected to decrease in temperature, decrease in the length of the body due to decrease in temperature will be $\alpha t L$. If this contract is prevented by applying external tensile force or by fixing the body to rigid supports, then tensile temp stress and tensile strain will be induced in the body

Example on temperature and thermal stresses and strain. 

1). A steel rod 15 m long is at a temperature of 15°C. Find the free expansion of the length when the temperature is raised to 65°C. Find the temperature stress produced when.

i) The expansion of the rod is prevented.

ii) The rod is permitted to expand by 6 mm.

Take $\alpha = 12 \times 10^{-6} /°C$.

$E = 200 \ GN/m^2$.

Soln :→ i) Free expansion of the rod.

$$\delta l = \alpha t l.$$

$$= \alpha (t_2 - t_1) L.$$

$$= 12 \times 10^{-6} (65 - 15) \times 15 \times 10^3.$$

$$\delta l = 9 \ mm.$$

ii) Expansion of the rod is prevented.

∴ ~~Compare~~ temperature stress $\sigma = \alpha t E$

$$\sigma = \alpha (t_2 - t_1) E$$

$$\sigma = 12 \times 10^{-6} (65 - 15) \times 200 \times 10^3$$

$$\sigma = 120 \ N/mm^2$$

iii) The rod is permitted to expand by 6mm. In this case the expansion is prevented

$= 9 - 6$

$= 3 mm.$

29

∴ strain

$$e = \frac{\text{Change in length or Expansion prevented}}{\text{original length}}$$

$$= \frac{3}{15 \times 10^3}$$

$$\boxed{e = 2 \times 10^{-4}}$$

∴ temperature stress $\sigma = e \times E$

$$= 2 \times 10^{-4} \times 200 \times 10^3$$

$$\boxed{\sigma = 40 \ N/mm^2}$$

Q) A steel rod 20 mm in diameter and 1.2 m long is heated through 120°C and at the same time subjected to a pull 'P'. If the total expansion of the rod is 3 mm, what should be the magnitude of 'P'? Take $\alpha_s = 12 \times 10^{-6} / °C$ and $E_s = 200$ GPa. (35)

Data given.

$d_s = 20$ mm.

$L_s = 1.2$ m $= 1.2 \times 10^3$ mm.

$t = 120°C$.

$\delta l = 3$ mm.

$\alpha_s = 12 \times 10^{-6} / °C$

$E_s = 200 \times 10^3$ N/mm$^2$.

To find $P = ?$

Soln:→

i) Elongation of the rod due to temperature

$$\delta l_1 = \alpha t L$$
$$= 12 \times 10^{-6} \times 120 \times 1.2 \times 10^3$$
$$\delta l_1 = 1.728 \text{ mm}.$$

ii) Elongation of the rod due to load P.

$$\delta l_2 = \frac{Pl}{AE}$$

$$\delta l_2 = \frac{P \times 1.2 \times 10^3}{\frac{\pi}{4}(20)^2 \times 200 \times 10^3}$$

$$\delta l_2 = 1.909 \times 10^{-5} P.$$

$\therefore$ Total elongation

$$\delta l = \delta l_1 + \delta l_2$$

$$3 = 1.728 + 1.908 \times 10^{-5} P$$

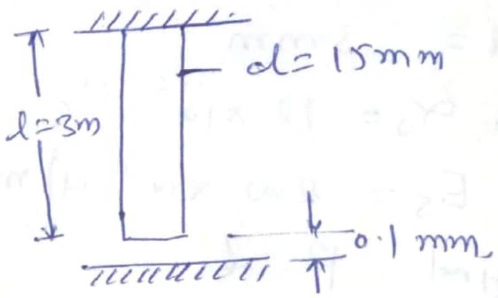
$$\therefore \boxed{P = 66603.83 \text{ N.}}$$

3). Determine the stresses in steel rod when the temperature increase through 40°C.

$\alpha = 12 \times 10^{-6} /°C$

$E = 200$ GPa.

$t = 40°C$.




Data given.

$\alpha = 12 \times 10^{-6} /°C$

$E = 200 \times 10^3 \text{ N/mm}^2$

$t = 40°C$

$d = 15$ mm.

$l = 3 \times 10^3$ mm.

To find $\sigma = ?$

Sol^n :-> The free expansion can be calculated by

$$\delta l = \alpha t L$$

$$\delta l = 12 \times 10^{-6} \times 40 \times 3000$$

$$\boxed{\delta l = 1.44 \text{ mm}}$$

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∴ the restricted change in length $= 1.44 - 0.1$
$= 1.34\ mm$ (3)

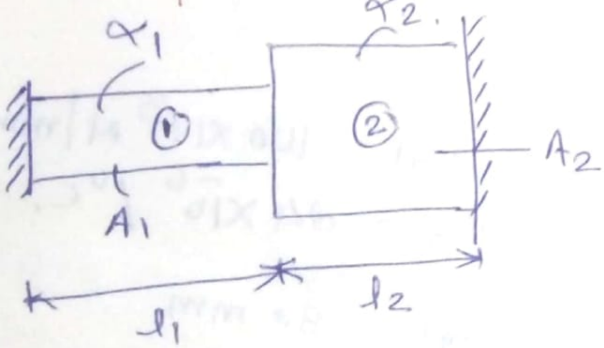
∴ ~~Restricted~~

$$\delta l = \frac{PL}{AE}$$

$$1.34 = \frac{\sigma \times 3000}{200 \times 10^3}$$

$$\boxed{\sigma = 89.33\ N/mm^2}$$

*) Metal placed in series.



i) force in ① = force in ②

$$(\sigma A)_1 = (\sigma A)_2$$

ii) free expansion $= \delta l_1 + \delta l_2$

$$= (\alpha t l)_1 + (\alpha t l)_2$$

iii) Restricted change in length $= \delta l_1 + \delta l_2$

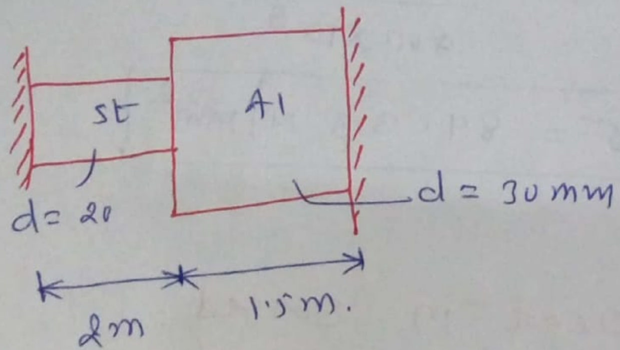
$$= \left(\frac{PL}{AE}\right)_1 + \left(\frac{PL}{AE}\right)_2$$

1) Determine the stress in steel and aluminium when the temperature increase by 40°c if

i) the support do not yield.
ii) If support yield by 0.1 mm.

$E_{st} = 200$ GPa, $\alpha_{st} = 12 \times 10^{-6}$ /°C

$E_{Al} = 100$ GPa, $\alpha_{Al} = 24 \times 10^{-6}$ /°C




Data given

$E_{st} = 200 \times 10^{3}$ N/mm$^2$    $E_{Al} = 100 \times 10^{3}$ N/mm$^2$

$\alpha_{st} = 12 \times 10^{-6}$ /°C    $\alpha_{Al} = 24 \times 10^{-6}$ /°C.

$d_{st} = 20$ mm.    $d_{Al} = 30$ mm.

$l_{st} = 2000$ mm.    $l_{Al} = 1500$ mm.

t = 40°C.

To find. $\sigma_{st} = ?$
$\sigma_{Al} = ?$

Sol$^n$:> As we know that
forces on steel = force on Al.

$$(\sigma A)_{st} = (\sigma A)_{Al}$$

$$\sigma_{st} \times \frac{\pi}{4} (d_{st})^2 = \sigma_{Al} \times \frac{\pi}{4} (d_{Al})^2$$

$$\sigma_{st} \times 314.16 = \sigma_{Al} \times (706.86)$$

$$\boxed{\sigma_{st} = 2.25\ \sigma_{Al}}$$

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Now Free expansion

$= (\delta l)_{st} + (\delta l)_{Al}$

$= (\alpha t l)_{st} + (\alpha t l)_{Al}$

$= (12 \times 10^{-6} \times 40 \times 2000) + (24 \times 10^{-6} \times 40 \times 1.5 \times 10^{3})$

$\boxed{\text{Free exp}^n = 2.4\,mm}$

case I) Support do not yield.

$\therefore \delta l = \delta l_{st} + \delta l_{Al}$

$2.4 = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{Al}$

$2.4 = \sigma_{st}\left(\frac{L}{E}\right)_{st} + \sigma_{Al}\left(\frac{L}{E}\right)_{Al}$

$2.4 = 2.25\,\sigma_{Al}\left(\frac{2000}{200 \times 10^{3}}\right) + \sigma_{Al}\left(\frac{1500}{100 \times 10^{3}}\right)$

$2.4 = 0.0225\,\sigma_{Al} + 0.015\,\sigma_{Al}$

$2.4 = 0.0375\,\sigma_{Al}$

$\therefore \boxed{\sigma_{Al} = 64\ N/mm^2}$

$\therefore \sigma_{st} = 2.25 \times \sigma_{Al}$

$\boxed{\sigma_{st} = 144\ N/mm^2}$

Case II) Support yield by 0.1 mm.

∴ Restricted change in length $= 2.4 - 0.1$
$= 2.3 mm$

$$\therefore (\delta l) = (\delta l)_{st} + (\delta l)_{Al}$$

$$2.3 = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{Al}$$

$$= 2.25\sigma_{Al}\left(\frac{2000}{210 \times 10^3}\right) + \sigma_{Al}\left(\frac{1500}{70 \times 10^3}\right)$$

$$2.3 = 0.0375\ \sigma_{Al}$$

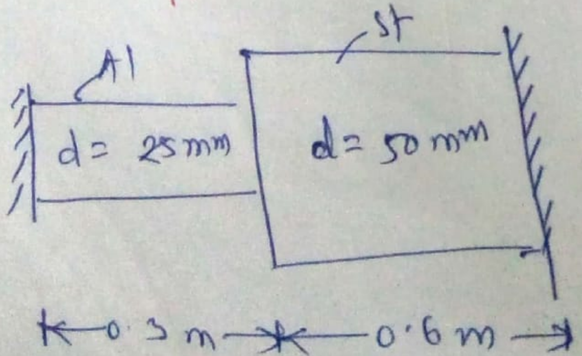
$$\therefore \boxed{\sigma_{Al} = 61.33\ N/mm^2}$$

$$\therefore \boxed{\sigma_{st} = 138\ N/mm^2}$$

H.W. A composite bar made up of Al and steel as place between support as shown in fig. bars are stress free at 40°C - what will be the stress at in the bar if

i) the support are not yield.

ii) The support are comes near by 0.1




$E_{st} = 210\ GPa$

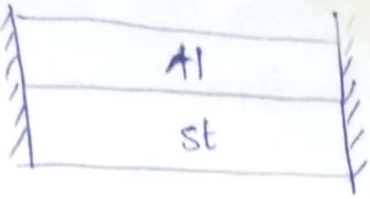
$E_{Al} = 74\ GPa$

$\alpha_{st} = 12 \times 10^{-6}$

$\alpha_{Al} = 23.4 \times 10^{-6}$

$t = 20°C$

Metals are in Parallel. (Temperature stresses).

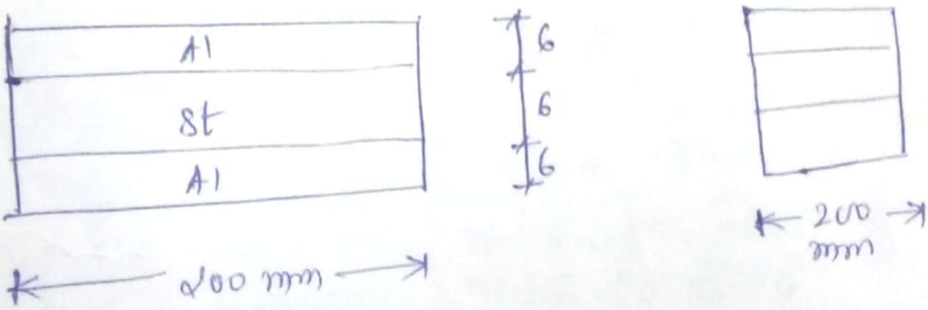



i) $(\sigma A)_{Al} = (\sigma A)_{st}$

ii) $e_{Al} + e_{st} = (\alpha t L)_{Al} - (\alpha t L)_{st}$

$(\because \alpha_{Al} > \alpha_{st})$.

* 1) A steel plate is placed between two aluminium plates are shown in fig. the ends are tightened together and temperature of assembly rise through 40°c determine stress in steel and aluminium.




$E_{st} = 210$ GPa.

$E_{Al} = 70$ GPa.

$\alpha_{Al} = 24 \times 10^{-6} /°C$

$\alpha_{st} = 12 \times 10^{-6} /°C$

Ans = $\sigma_{Al} = 4032\ N/mm^2$

$\sigma_{st} = 8064\ N/mm^2$

Longitudinal or
Linear strain :→ The strain in the direction of applied force is known as linear or longitudinal strains. It is denoted by: e

∴ linear or longitudinal strain $e = \frac{\delta L}{L}$.

lateral strain :→ strain in the direction right angle to the direction of applied force is known as lateral strain or secondary strain.

$$\therefore \text{Lateral strain} = \frac{\text{change in lateral dimension}}{\text{original lateral dimension}}.$$

∴ for rectangular bar

$$\text{lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta t}{t}$$

for circular bar.

$$\text{lateral strain} = \frac{\delta d}{d}.$$

Poisson's Ratio :- When a homogeneous material is loaded within its elastic limit, the ratio of the lateral strain to linear strain is constant, and that constant is known as poisson's ratio. It is denoted by $\mu$ or $\frac{1}{m}$.

the value of m lies between 3 and 4 for most metals. i.e poisson's ratio for most of the metal lies between 0.25 to 0.34.

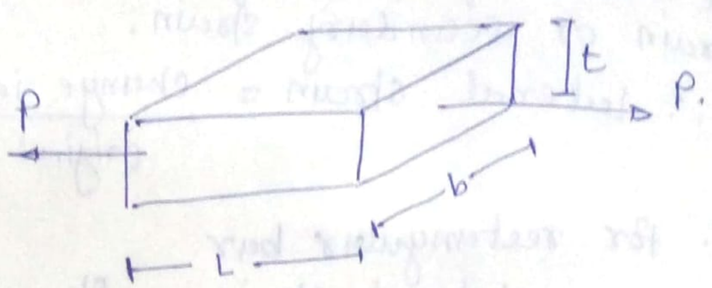
$$\therefore \text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{Linear strain}}$$

$$\therefore \mu = \frac{\text{lateral strain}}{e}$$

$$\therefore \text{lateral strain} = e \times \mu$$

$$\text{or lateral strain} = e \times \frac{1}{m}$$

Consider a metallic bar of length 'L', width 'b' and thickness 't' subjected to an axil tensile force 'p' as shown in fig. Due to tensile load P, the length of the bar is increases by $\delta l$. This increase in length is associated with the decrease in lateral side 'b' and 't'. Therefore the linear strain is positive and lateral strain are negative.




$\therefore$ linear strain $e = \frac{\delta L}{L}$

and lateral strain $= \frac{-\delta b}{b} = \frac{-\delta t}{t}$

$\therefore$ We can say that

$$\boxed{\text{lateral strain} = -\mu \times e.}$$

Note :- If the same bar is subjected to axial compressive load P, there will be decreasing in length which will be followed by increasing in lateral dimension 'b' and 't'. In this case, the linear strain is negative and lateral strains are positive.

*) Volumetric strain :- When a body is subjected to external forces on its faces, there will be change in volume. The ratio of change in volume to original volume is known as volumetric strain. It is denoted by $e_v$.

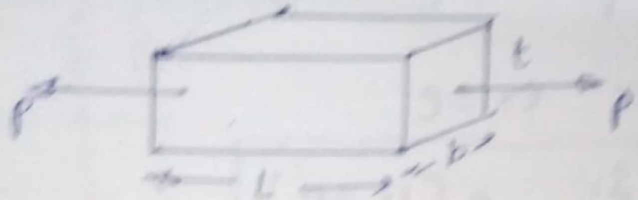
$\therefore e_v = \frac{\delta V}{V_o}$

Volumetric strain is the algebric sum of all and linear strain i.e $e_v = e_x + e_y + e_z$.

## Uni-Axial loading.

If the force is applied in any one direction, say x-direction, it is called uni-axial and the body is said to be under the action of uni-axial stress system.

Consider a rectangular bar of length 'L' width 'b' and thickness 't' subjected to a tensile force P along x-direction



Stress in x-direction $\sigma_x = \frac{P}{A} = \frac{P}{b \times t}$

$\because$ there is no load in y and z direction

$\therefore$ The stresses in y and z directions are zero.

$\therefore \sigma_y = \sigma_z = 0.$

Strain in x-direction due to P is linear strain.

$e_x = \frac{\sigma_x}{E}$ $\quad (\because E = \frac{\sigma}{e})$

strain in y and z-direction are lateral strain.

$\therefore e_y = e_z = -\mu \times \text{linear strain}$

$= -\mu \times e$

$e_y = e_z = -\mu \times \frac{\sigma_x}{E}$

$\therefore$ total volumetric strain

$\frac{\delta V}{V} = e_x + e_y + e_z$

$= \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E}$

$$\therefore \frac{\delta v}{V} = \frac{\sigma_x}{E}(1-2\mu)$$

But $\frac{\sigma_x}{E} = e_x$

$$\therefore \frac{\delta v}{V} = e_x(1-2\mu)$$

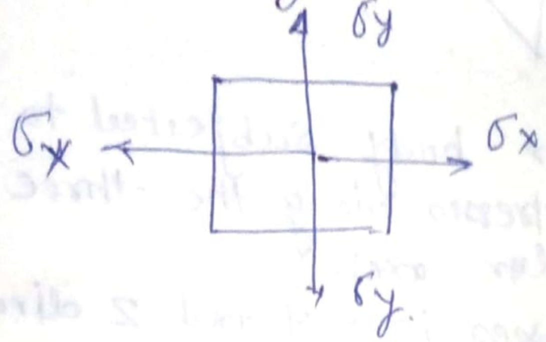
or $e_x = e$

$$\therefore \boxed{\frac{\delta v}{V} = e(1-2\mu)}$$

## Concept of Bi-axial loading.

If the forces are applied in two mutually perpendicular directions say x and y, they are called bi-axial forces and the body is said to be under the action of bi-axial stress system.

Consider a pieces of material subjected to the two direct tensile stresses along the two mutually perpendicular axes.




Let $\sigma_x$ = stress in x-direction  $e_x$ = strain in x-direction.

$\sigma_y$ = stress in y-direction  $e_y$ = strain in y-direction.

∴ strain in x direction.

i) Due to $\sigma_x$ alone, $e_x = \frac{\sigma_x}{E}$

ii) Due to $\sigma_y$ alone, $e_x = -\mu \frac{\sigma_y}{E}$

∴ total strain in x-direction

$$\boxed{e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}}$$

Strain in y-direction.

i) Due to $\sigma_x$ alone, $e_y = -\mu \frac{\sigma_x}{E}$

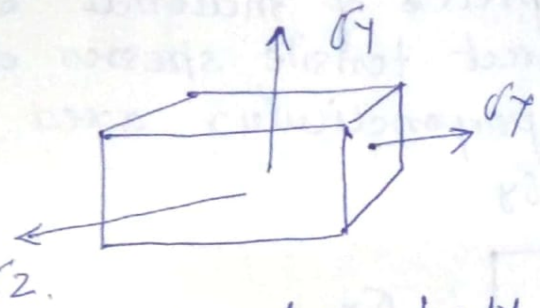
ii) Due to $\sigma_y$ alone, $e_y = \frac{\sigma_y}{E}$

∴ total strain in y-direction.

$$\boxed{e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E}}$$

Tri- axial loading.

If the force is acting in three mutually perpendicular direction x, y, z, they called tri-axial forces and the body is said to be under the action of tri-axial stress system.



Consider a rectangular body subjected to the three direct tensile stresses along the three mutually perpendicular axes.

$\sigma_x$, $\sigma_y$, and $\sigma_z$ = stresses in x, y and z direction respectively.

$e_x, e_y$ and $e_z$ = strain in x, y, and z- directions resp.

*) Strain in x direction

i) Due to $\sigma_x$ alone $e_x = \frac{\sigma_x}{E}$

ii) Due to $\sigma_y$ alone, $e_x = -\mu \frac{\sigma_y}{E}$

iii) Due to $\sigma_z$ alone $e_x = -\mu \frac{\sigma_z}{E}$

∴ Total strain in x direction.

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

similarly, total strain in y- direction.

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_x}{E}$$

and total strain in z- direction.

$$e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E}$$

∴ the volumetric strain

$$\frac{\delta v}{v} = e_x + e_y + e_z$$

$$\frac{\delta v}{V} = \frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E} - 2\mu\frac{\sigma_x}{E} - 2\mu\frac{\sigma_y}{E} - 2\mu\frac{\sigma_z}{E}$$

$$= \left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right) - 2\mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$\boxed{\frac{\delta v}{V} = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)}$$

## *) Bulk Modulus (K)

When a body is subjected to three mutually perpendicular like stresses of same intensity, then the ratio of direct stress to the corresponding volumetric strain of the body is constant and is known as Bulk modulus. It is denoted by K.

$$\therefore \text{Bulk modulus } K = \frac{\text{direct stress}}{\text{Volumetric strain}}$$

$$\therefore K = \frac{\sigma}{\frac{\delta v}{V}}$$

Imp :- Relation Between Modulus of Elasticity (E) and Bulk modulas (K)

⇒ We know that a body is subjected to a tri-axial stress system, its volumetric strain is given by

$$\frac{\delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$

then

$$\frac{\delta V}{V} = \frac{\sigma + \sigma + \sigma}{E}(1-2\mu)$$

$$\frac{\delta V}{V} = \frac{3\sigma}{E}(1-2\mu)$$

But $K = \dfrac{\sigma}{\frac{\delta V}{V}}$

$$\therefore K = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$K = \frac{E}{3(1-2\mu)}$$

$$\boxed{E = 3K(1-2\mu)}$$

# Relation Between, E, G and K,

⇒ We know that,

$$E = 2G(1+\mu) \quad \text{--- (i)}$$

and $E = 3K(1-2\mu) \quad \text{--- (ii)}$

from equation (1)

$$1+\mu = \frac{E}{2G}.$$

$$\mu = \frac{E}{2G} - 1$$

put the value of $\mu$ in eqn (2)

$$\therefore \quad E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]$$

$$E = 3K\left[1 - \frac{E}{G} + 2\right]$$

$$E = 3K\left(3 - \frac{E}{G}\right)$$

$$E = 9K - \frac{3KE}{G}$$

$$E = \frac{9KG - 3KE}{G}$$

$$EG = 9KG - 3KE$$

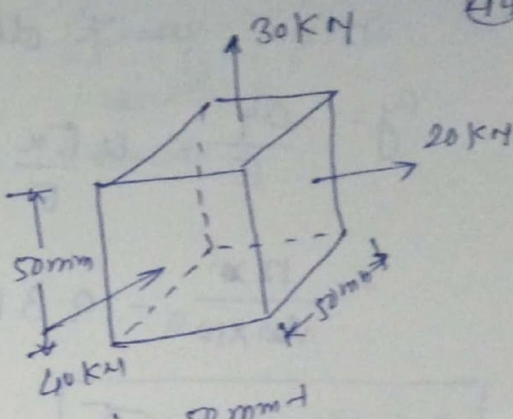
$$EG + 3KE = 9KG.$$

$$E(3K+G) = 9KG$$

$$\boxed{E = \frac{9KG}{3K+G}}$$

). Determine
i) Change in length along x, y, z axis
ii) change in volume.

$m = \frac{10}{3}$ and $E = 200$ GPa.




Data given.

$P_x = 20$ KN. $\quad l = 50$ mm

$P_y = 30$ KN. $\quad b = 50$ mm

$P_z = 40$ KN. $\quad t = 50$ mm.

$m = \frac{10}{3}$

$\therefore \mu = \frac{1}{m} = \frac{3}{10} = 0.3.$

$E = 200$ GPa $= 200 \times 10^3$ N/mm$^2$.

To find. $\delta l_x, \delta l_y, \delta l_z$ and $\frac{\delta v}{v}$.

Soln:→ stress in X-direction.

$$\sigma_x = \frac{P_x}{A} = \frac{20 \times 10^3}{50 \times 50} = 8 \text{ N/mm}^2.$$

$$\sigma_y = \frac{P_y}{A} = \frac{30 \times 10^3}{50 \times 50} = 12 \text{ N/mm}^2$$

$$\sigma_z = \frac{P_z}{A} = -\frac{40 \times 10^3}{50 \times 50} = -16 \text{ N/mm}^2$$

(∴ − sing for compressive loading).

Now strain in X - direction.

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$e_x = \frac{8}{200 \times 10^3} - 0.3 \times \frac{12}{200 \times 10^3} - 0.3\left(-\frac{16}{200 \times 10^3}\right)$$

$$\boxed{e_x = 4.60 \times 10^{-5}}$$

strain in y- direction.

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{12}{200\times10^3} - 0.3\left(\frac{8}{200\times10^3}\right) - 0.3\left(-\frac{16}{200\times10^3}\right)$$

$$\boxed{e_y = 7.2\times10^{-5}}$$

strain in z- direction.

$$e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$= \left(-\frac{16}{200\times10^3}\right) - 0.3\left(\frac{8}{200\times10^3}\right) - 0.3\left(\frac{12}{200\times10^3}\right)$$

$$\boxed{e_z = -1.1\times10^{-4}.}$$

∴ Volumetric strain.

$$e_v = e_x + e_y + e_z$$

$$= 4.60\times10^{-5} + 7.20\times10^{-5} - 1.1\times10^{-4}$$

$$\boxed{e_v = 8\times10^{-6}}$$

Now change in length in x- direction.

$$e_x = \frac{\delta l_x}{L_x}$$

$$4.60\times10^{-5} = \frac{\delta l_x}{50}$$

$$\therefore \boxed{\delta l_x = 2.3\times10^{-3}\,mm.}$$

change in length in y- direction. (change in ~~width~~ thickness)

$$e_y = \frac{\delta l_y}{l_y} = \frac{\delta t}{t}.$$

$$7.2 \times 10^{-5} = \frac{\delta l_y}{50}$$

$$\therefore \boxed{\delta l_y = 3.6 \times 10^{-3}\ mm}$$

change in length in z-direction. (change in ~~thickness~~ width)

$$e_z = \frac{\delta l_z}{l_z} = \frac{\delta b}{b}.$$

$$-1.1 \times 10^{-4} = \frac{\delta l_z}{50}$$

$$\boxed{\delta l_z = -5.5 \times 10^{-3}\ mm}$$

$\therefore$ change in volume.

$$e_v = \frac{\delta V}{V}.$$

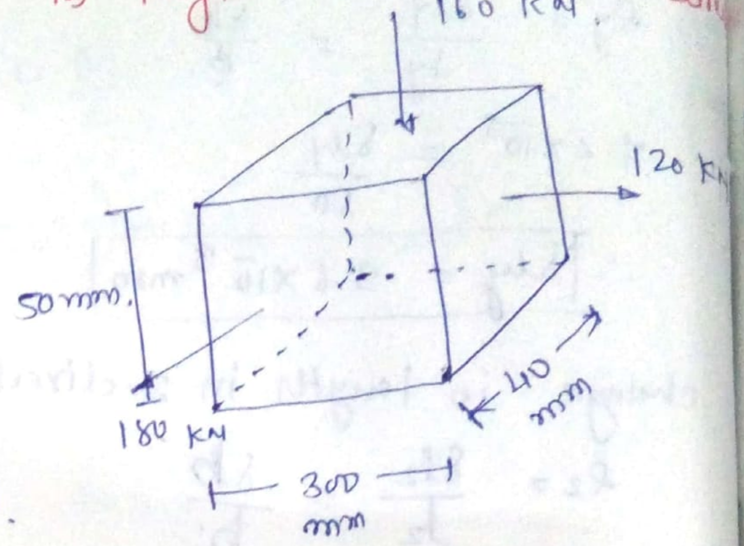
$$8 \times 10^{-6} = \frac{\delta V}{50 \times 50 \times 50}$$

$$\therefore \boxed{\delta V = 1\ mm^3}$$

2). Determine the change in volume for given load condition. and change in length in all three dir

$E = 200$ GPa.

$\mu = 0.25$




Data given.

$P_x = 120$ KN $= 120 \times 10^3$ N.

$P_y = 160$ KN $= 160 \times 10^3$ N.

$P_z = 180$ KN $= 180 \times 10^3$ N.

$l = 300$ mm.

$b = 40$ mm

$t = 50$ mm.

$E = 200$ GPa $= 200 \times 10^3$ N/mm$^2$

$\mu = 0.25$

To find. $\delta v = ?$
$\delta l_x = ?$ $\delta l_y = ?$ $\delta l_z = ?$

Soln :→ stress in x-direction.

$$\sigma x = \frac{Px}{Ax} = \frac{Px}{b \times t} = \frac{120 \times 10^3}{40 \times 50}$$

$$\boxed{\sigma x = 60 \text{ N/mm}^2}$$

stress in y-direction

$$\sigma y = \frac{Py}{A_y} = \frac{-Py}{L \times b} = \frac{-160 \times 10^3}{300 \times 40}$$

$$\boxed{\sigma y = -13.33 \text{ N/mm}^2}$$

$\therefore \sigma y = 13.33$ N/mm$^2$ (compressive) stress in z-direction

$$\sigma_z = \frac{P_z}{A_z} = \frac{180 \times 10^3}{300 \times 50} = 12 \ N/mm^2$$

Now strain in x-direction.

$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{60}{200 \times 10^3} - 0.25\left(\frac{-13.33}{200 \times 10^3}\right) - 0.25\left(\frac{12}{200 \times 10^3}\right)$$

$$\boxed{e_x = 3.26 \times 10^{-4}} \quad 3.01 \times 10^{-4}$$

strain in y-direction.

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{-13.33}{200 \times 10^3} - 0.25\left(\frac{60}{200 \times 10^3}\right) - 0.25\left(\frac{12}{200 \times 10^3}\right)$$

$$\boxed{e_y = -1.56 \times 10^{-3}} = -1.566 \times 10^{-4}$$

strain in z-direction.

$$e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$= \frac{12}{200 \times 10^3} - 0.25\left(\frac{60}{200 \times 10^3}\right) - 0.25\left(\frac{-13.33}{200 \times 10^3}\right)$$

$$\boxed{e_z = -8.73 \times 10^{-4}} \quad -1.66 \times 10^{-6}$$

Now Total volume or volumentic strain.

$$e_v = e_x + e_y + e_z$$

$$= 3.26 \times 10^{-4} - 1.56 \times 10^{-3} - 8.73 \times 10^{-4}$$

$$\boxed{e_v = -2.10 \times 10^{-3}} = 1.446 \times 10^{-4}$$

$\therefore$ change in volume

$$e_v = \frac{\delta v}{V}$$

$$-2.10 \times 10^{-3} = \frac{\delta v}{(50 \times 300 \times 40)}$$

$$\boxed{\delta v = -1260 \text{ mm}^2} \times \quad 88.01 \text{ mm}^3$$

Now change in length in x-direction

$$e_x = \frac{\delta l_x}{l_x}$$

$$3.26 \times 10^{-4} = \frac{\delta l_x}{300}$$

$$\boxed{\delta l_x = 0.01304 \text{ mm}} \times = 0.0904 \text{ mm}.$$

change in length in y-direction. (change in width)

$$e_y = \frac{\delta l_y}{l_y}$$

$$-1.56 \times 10^{-3} = \frac{\delta l_y}{50}$$

$$\boxed{\delta l_y = -0.078 \text{ mm}} \times \quad -7.83 \times 10^{-3} \text{ mm}$$

Change in length in z-direction. ($\therefore$ change in thickness)

$$e_z = \frac{\delta l_z}{l_z}$$

$$-8.73 \times 10^{-4} = \frac{\delta l_z}{40}$$

$$\boxed{\delta l_z = -0.2619 \text{ mm}} = 6.664 \times 10^{-5} \text{ mm}$$

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